MATH/CSCI 2112, DISCRETE STRUCTURES I, FALL 2005
Handout 6: Rules for Program Correctness Proofs

Precondition strengthening

$$\frac{P_1 \Rightarrow P \quad \{P\}\, C\, \{Q\}}{\{P_1\}\, C\, \{Q\}}$$

Postcondition weakening

$$\frac{\{P\}\, C\, \{Q\} \quad Q \Rightarrow Q_1}{\{P\}\, C\, \{Q_1\}}$$

Conjunction rule

$$\frac{\{P_1\}\, C\, \{Q_1\} \quad \{P_2\}\, C\, \{Q_2\}}{\{P_1 \wedge P_2\}\, C\, \{Q_1 \wedge Q_2\}}$$

Disjunction rule

$$\frac{\{P_1\}\, C\, \{Q_1\} \quad \{P_2\}\, C\, \{Q_2\}}{\{P_1 \vee P_2\}\, C\, \{Q_1 \vee Q_2\}}$$

Sequencing rule (concatenation)

$$\frac{\{P\}\, C_1\, \{Q\} \quad \{Q\}\, C_2\, \{R\}}{\{P\}\ C_1; C_2\, \{R\}}$$

Modified concatenation rule

$$\frac{\{P\}\, C_1\, \{Q\} \quad Q \Rightarrow R \quad \{R\}\, C_2\, \{S\}}{\{P\}\ C_1; C_2\, \{S\}}$$

Assignment rule for variables

$$\frac{}{\{Q[E/x]\}\, x := E\, \{Q\}}$$

Modified assignment rule for variables

$$\frac{}{\{E \text{ well-defined}, Q[E/x]\}\, x := E\, \{Q\}}$$

If-then-else rule, forward

$$\frac{\{P \wedge B\}\, C_1\, \{Q\} \quad \{P \wedge {\sim}B\}\, C_2\, \{R\}}{\{P\}\, \textbf{if } B \textbf{ then } C_1 \textbf{ else } C_2\, \{Q \vee R\}}$$

If-then-else rule, backward

$$\frac{\{P\}\, C_1\, \{Q\} \quad \{R\}\, C_2\, \{Q\}}{\{(P \wedge B) \vee (R \wedge {\sim}B)\}\, \textbf{if } B \textbf{ then } C_1 \textbf{ else } C_2\, \{Q\}}$$

If-then-else rule, both ways

$$\frac{\{P \wedge B\}\, C_1\, \{Q\} \quad \{P \wedge {\sim}B\}\, C_2\, \{Q\}}{\{P\}\, \textbf{if } B \textbf{ then } C_1 \textbf{ else } C_2\, \{Q\}}$$

If-then rule, both ways

$$\frac{\{P \wedge B\}\, C_1\, \{Q\} \quad P \wedge {\sim}B \Rightarrow Q}{\{P\}\, \textbf{if } B \textbf{ then } C_1\, \{Q\}}$$

While rule

$$\frac{\forall k.(\{I(k) \wedge B\}\, C\, \{I(k+1)\}) \quad P \Rightarrow I(0) \quad I(k) \wedge {\sim}B \Rightarrow Q \quad \exists k.(I(k) \Rightarrow {\sim}B)}{\{P\}\, \textbf{while } B \textbf{ do } C\, \{Q\}}$$

Problem 1. In each of parts (a)–(e), find the weakest precondition P that makes the given Hoare triple correct. Simplify your answers (logically and algebraically) where applicable.

(a) $\{P\}\, \mathtt{x} := 5 - \mathtt{x}\, \{n = x + 7\}$

(b) $\{P\}\, \mathtt{a} := -\mathtt{a};\ \mathtt{n} := \mathtt{n} + \mathtt{a}\, \{n \neq a \wedge n \neq 0\}$

(c) $\{P\}\, \mathtt{a} := \mathtt{a} + \mathtt{b};\ \mathtt{b} := \mathtt{a} - \mathtt{b};\ \mathtt{a} := \mathtt{a} - \mathtt{b}\, \{a \geqslant b + 2\}$

(d) $\{P\}\, \textbf{if } \mathtt{x} > \mathtt{y} \textbf{ then } \mathtt{x} := \mathtt{x} + 2 \textbf{ else } \mathtt{y} := \mathtt{y} - 2\, \{x \neq y\}$

(e) $\{P\}\, \textbf{if } \mathtt{x} > \mathtt{y} \textbf{ then } \mathtt{z} := \mathtt{x} \textbf{ else } \mathtt{z} := \mathtt{y}\, \{z > y\}$