# Schedule for
# Combinatorial Algebra meets Algebraic Combinatorics

**All the talks are in Jeffrey Hall, Room 225**

FRIDAY JANUARY 22

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|---|---|---|
| 2:30 | – 3:30 | *Colloquium: Sara Faridi* |
| | | COLLOQUIUM SNACKS |
| 4:30 | – 5:30 | *Mike Zabrocki (I)* |
| 5:45 | – 6:15 | *Yong Su Shin* |
| 7:30 | | DINNER AT KINGSTON BREW PUB (34 CLARENCE ST.) |

SATURDAY JANUARY 23

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|---|---|---|
| 8:00 | | COFFEE AND SNACKS |
| 8:30 | – 9:30 | *Mike Zabrocki (II)* |
| 9:45 | – 10:45 | *Jennifer Morse* |
| | | COFFEE BREAK |
| 11:15 | – 11:45 | *Tony Iarrobino* |
| 12:00 | – 12:45 | *Mats Boij (I)* |
| | | LUNCH BREAK |
| 2:00 | – 3:00 | *Christophe Reutenauer* |
| | | COFFEE BREAK |
| 3:30 | – 4:30 | *Nantel Bergeron* |
| 4:45 | – 5:15 | *Adam Van Tuyl* |
| 6:00 | | DINNER AT AROMA RESTAURANT (248 ONTARIO ST.) |
| 8:30 | | PARTY AT TONY GERAMITA'S HOUSE |

SUNDAY JANUARY 24

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|---|---|---|
| 8:15 | | COFFEE AND SNACKS |
| 8:45 | – 9:15 | *Li Li* |
| 9:30 | – 10:00 | *Christian Stump* |
| | | COFFEE BREAK |
| 10:30 | – 11:00 | *Weidong Gao* |
| 11:15 | – 12:00 | *Mats Boij (II)* |

# Titles & Abstracts

1. **Nantel Bergeron (York University):** *Non-commutative combinatorial inverse systems*

   **Abstract:** We introduce a notion of a combinatorial inverse system in non-commutative variables.We present two important examples, some conjectures and results. These conjectures and results were suggested and supported by computer investigations.

   This is joint work with H. Li and J. C. Aval.

2. **Mats Boij (KTH, Sweden):** *The cone of Betti diagrams I & II*

   **Abstract:** Working on the Multiplicity conjecure, Jonas Söderberg and I came up with a set of conjectures on the set of Betti diagrams of graded Cohen-Macaulay modules up to scaling. The main idea is that Betti diagrams can be decomposed into sums of pure diagrams, with possibly rational entries, in a very precise manner. In 2007 part of the conjectures were proven by Eisenbud, Fløystad and Weyman and soon after that they were completely proven by Eisenbud and Schreyer. In their work they also discovered a beautiful connection and duality with the same problem for cohomology tables of vector bunndles on projective spaces. It is interesting to note that the combinatorial structure of the cone as a simplicial fan plays an important role. Later these results have been extended to the non-Cohen-Macaulay case and on the dual side to coherent sheaves.

   In the first talk I will give the background and explain the known results so far. In the second talk, I will discuss some applications of the results and I will show how the use of scaling can help in studying Betti diagrams and Hilbert functions. Among the applications I will talk about the parameter spaces of modules with a given Hilbert function.

3. **Sara Faridi (Dalhousie University):** *Graphs, hypergraphs and algebras associated with them* (Colloquium Talk)

   **Abstract:** This talk explores how algebraists have been using graphs and their higher-dimensional counterparts to discover large classes of algebras having various algebraic properties. Conversely, combinatorial descriptions of subclasses of hypergraphs can be given using these algebraic properties. The talk will highlight the contributions of many people at different points of time and from different points of view.

4. **Weidong Gao (Nankai University):** *Some combinatorial problems and group algebras*

   **Abstract:** In this talk, we shall present several recent results in combinatorics obtained by employing group algebras as a tool in a similar way.

5. **Tony Iarrobino (Northeastern University):** *Commuting nilpotent matrices and standard bases for ideals*

   **Abstract:** Fix an $n \times n$ nilpotent matrix $B$ whose Jordan blocks are given by the partition $P$ of $n$. Consider the ring $C_B \subset M_n(k)$ of $n \times n$ matrices with entries in an algebraically closed field $k$ that commute with $B$, and its subset $N_B$ of nilpotent matrices. Then $N_B$ is an irreducible algebraic variety: so there is a Jordan block partition $Q(P)$ of the generic matrix $A \in N_B$. What is $Q(P)$? Various groups including P. Oblak with T. Kosir, D.I. Panyushev, and R. Basili and the speaker have posed and worked on this problem. We report on this problem, and our approach using standard bases for ideals of the ring $k\{x, y\}$. This is joint work with R. Basili and L. Khatami.

6. **Li Li (University of Illinois at Urbana-Champaign):** *On the algebra and combinatorics of q,t-Catalan numbers*

   **Abstract:** Haiman proved that the $q,t$-Catalan number is the Hilbert series of the graded vector space $M = \oplus M_{d_1,d_2}$ spanned by a minimal set of generators for the ideal of the diagonal locus of $(C^2)^2$. It is natural to ask for a combinatorial construction of such generators. In this talk we give upper bounds for the dimension of $M_{d_1,d_2}$ in terms of partition numbers, and find all bi-degrees $(d_1, d_2)$ that achieve equality. For these bi-degrees, we answer the aforementioned question. This is joint work with Kyungyong Lee.

7. **Jennifer Morse (Drexel University):** *Affine Schubert Calculus and K-theory*

   **Abstract:** Classical Schubert calculus addressed enumerative problems in projective geometry by converting them into problems of computation in the cohomology ring of the Grassmannian. Combinatorial Schur theory has since transformed Schubert calculus into a contemporary theory tied to problems in representation theory and physics.

   We will illustrate how this comes about with a concrete example. We will then see how our work with a new family of polynomials has led to a similar transformation of the modern theory of "affine Schubert calculus", now involving the type-A affine Weyl group, Macdonald polynomials, and Gromov-Witten invariants.

   Time permitting, we will discuss the applications of our work to affine K-theory.

   No background in geometry, Schubert calculus, or Macdonald polynomials will be assumed.

8. **Christophe Reutenauer (UQAM):** *$SL_2$-tilings*

   **Abstract**: Call *$SL_2$-tiling* a filling of the discrete plane by elements of a ring (the coefficients) in such a way that each connected 2 by 2 submatrix has determinant 1 (similar objects have been studied by Coxeter and Conway; they call them *frieze-patterns* and, in the present setting, should be called *partial* $SL_2$-tilings).

   Given a bi-infinite word on $\{x, y\}$, interpreted as a path in the discrete plane, called the *frontier*, put 1's at its vertices. Then one may uniquely complete this picture in a $SL_2$-tiling; it turns out that the coefficients of the tiling are all positive integers; this is explained by matrix product formulas for these coefficients (the fact that these coefficients are integers also follows from the *Laurent phenomenon* studied by Fomin and Zelevinsky in the framework of their theory of cluster algebras).

   It turns out that the geometry of the frontier implies the existence of perfect squares in the tiling: in some diagonal appear the squares of the numbers which appear in some row, or column (see below).

   Also, if the frontier is periodic, the sequences on each diagonal of the tiling are the coefficients of an N-*rational* series (that is, a positively rational series; in other words, the generating series of some *rational language*); an example is given by the tiling associated to the alternate frontier ...$xyxyxyxy$...., where appear the Fibonacci numbers of even rank.

   Motivations of these constructions are the so-called *frises*, associated to acyclic digraphs (this construction also comes from the theory of cluster algebras). In a joint work with I. Assem and D. Smith, we showed that the sequences of the frise are all rational if and only if the digraph is a Dynkin diagram, or an affine diagram, with an acyclic orientation (recall that these diagrams play an important role in the classification of Lie algebras, and other mathematical structures).

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| | | | | | | | | | **1** | 3 | 8 | 13 | 18 | |
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| | | | | | | | | **1** | 2 | 9 | $5^2$ | 41 | 57 | |
| | | | | | | | | **1** | 3 | 14 | 39 | $8^2$ | 89 | |
| | | | **1** | **1** | **1** | **1** | **1** | **1** | 4 | 19 | 53 | 87 | $11^2$ | |
| **1** | **1** | **1** | **1** | 2 | 3 | 4 | 5 | 6 | $5^2$ | 119 | 332 | 545 | 758 | |
| **1** | 2 | 3 | 4 | 9 | 14 | 19 | 24 | 29 | 121 | $24^2$ | 1607 | 2368 | 3669 | ... |
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9. **Yong-Su Shin (Sungshin Women's University, Korea):** *Some applications when Green's upper bound holds*

   Joint work with Jeaman Ahn (Kongju National University, Korea) and Anthony V. Geramita (Queen's and Genoa)

   **Abstract:** Macaulay [2] discovered the rule governing the growth of such functions. He expressed that rule in terms of certain expansions of the values of these functions by binomial numbers. Green's approach to Macaulay's Theorem included a brand new element – a comparison between the Hilbert function of a variety and that of its general hyperplane section [1].

   It is well known that the Hilbert function and Hilbert polynomial of an embedded algebraic variety, although being natural algebraic invariants associated to the coordinate ring of the embedded variety, also carry significant geometric information about the variety - some of the information being connected to the embedding, like the degree of the variety, while other information is more intrinsic (i.e., does not depend on the embedding) like the dimension and genus of the variety.

   If $\mathbb{X}$ is an irreducible variety in $\mathbf{P}^n$ then the number of binomial summands that (eventually) appear in the Hilbert function of $\mathbb{X}$ is an invariant of $\mathbb{X}$ denoted $G(\mathbb{X})$ and called the *Gotzmann number of* $\mathbb{X}$. It is not difficult to show that $\deg \mathbb{X} \leq G(\mathbb{X})$. We characterize the varieties $\mathbb{X}$ for which this inequality is an equality. This result follows from a detailed investigation of precisely when the inequality in Green's Hyperplane Restriction Theorem is an equality.

   If we denote by $M(\mathbb{X})$ the least integer such that the inequality in Green's Theorem is an equality for all $d \geq M(\mathbb{X})$ then one easily sees that $M(\mathbb{X}) \leq G(\mathbb{X}) + 1$. We improve this to show that $M(\mathbb{X}) \leq G(\mathbb{X})$ and then go on to show that $M(\mathbb{X}) = G(\mathbb{X})$ or $M(\mathbb{X}) = 1$. Connecting this to our earlier geometric discussion we show that if $\mathbb{X}$ is a reduced, equidimensional closed subscheme of $\mathbf{P}^n$ then either $G(\mathbb{X}) = \deg(\mathbb{X})$ or $G(\mathbb{X}) = M(\mathbb{X})$.

[1] M. Green. Restrictions of linear series to hyperplanes, and some results of Macaulay and Gotzmann. In *Algebraic curves and projective geometry (Trento, 1988)*, volume 1389 of *Lecture Notes in Math.*, pages 76–86. Springer, Berlin, 1989.

[2] F.S. Macaulay. Some properties of enumeration in the theory of modular systems. *Proc. Lond. Math. Soc.*, 26(1):531–555, 1927.

10. **Christian Stump (UQAM):** *q,t-Fuss-Catalan numbers for finite reflection groups*

    **Abstract:** In type A, the $q,t$-Fuss-Catalan numbers can be defined as a bigraded Hilbert series of a module associated to the symmetric group. We generalize this construction to (finite) complex reflection groups, exhibit several conjectured algebraic and combinatorial properties, and investigate the cases of dihedral and cyclic groups.

11. **Adam Van Tuyl (Lakehead University):** *A conjecture about colouring graphs and connections to associated primes of ideals*

    **Abstract:** In this talk, I will introduce a conjecture about critically chromatic graphs. I will explain why a solution to this conjecture would show that the associated primes of square-free monomial ideals of height two have the persistence property. This is joint work with C.A. Francisco and H.T. Ha.

12. **Mike Zabrocki (York University):**

    - *Combinatorial Hopf Algebras I: a zoo of Hopf algebras*

      **Abstract:** I will present an overview of what Combinatorial Hopf Algebras (CHAs) are about by introducing definition and examples. I will try to show how examples of CHAs are related to each other and where they can appear in the literature before they are recognized as CHAs. I will also give some examples where these objects appear in other areas of mathematics.

    - *Combinatorial Hopf Algebras II: the wilderness of Hopf algebras*

      **Abstract:** I will present some directions of research related to CHAs motivated classical problems. I will try to relate them to examples and indicate what we expect to gain by studying them.