$$\begin{array}{|c|c|c|c|} \hline 1&2&3&4\\ \hline \end{array} < \begin{array}{|c|c|c|} \hline 1&2&3\\ \hline 4\\ \cline{1-1} \end{array} < \begin{array}{|c|c|c|} \hline 1&2&4\\ \hline 3\\ \cline{1-1} \end{array} < \begin{array}{|c|c|c|} \hline 1&3&4\\ \hline 2\\ \cline{1-1} \end{array} < \begin{array}{|c|c|} \hline 1&2\\ \hline 3&4\\ \hline \end{array}$$

$$< \begin{array}{|c|c|} \hline 1&3\\ \hline 2&4\\ \hline \end{array} < \begin{array}{|c|c|} \hline 1&2\\ \hline 3\\ \cline{1-1} 4\\ \cline{1-1} \end{array} < \begin{array}{|c|c|} \hline 1&3\\ \hline 2\\ \cline{1-1} 4\\ \cline{1-1} \end{array} < \begin{array}{|c|c|} \hline 1&4\\ \hline 2\\ \cline{1-1} 3\\ \cline{1-1} \end{array} < \begin{array}{|c|} \hline 1\\ \hline 2\\ \hline 3\\ \hline 4\\ \hline \end{array}$$

Monomials:

$$x_{T_1}^{T_1} = 1 \qquad x_{T_2}^{T_2} = x_3 \qquad x_{T_2}^{T_3} = x_2x_3 \qquad x_{T_3}^{T_2} = x_2 \qquad x_{T_3}^{T_3} = x_2x_3 \qquad x_{T_4}^{T_4} = x_2x_3^2$$

Young Symmetrizers:

$$\varepsilon_{T_1} = (1) = \sigma_{T_1}$$

$$\varepsilon_{T_2} = (1) + (1,2) - (1,3) - (1,2,3) \qquad \sigma_{T_2} = (1) + (1,2) - (1,3) - (1,3,2)$$

$$\varepsilon_{T_3} = (1) + (1,3) - (1,2) - (1,3,2) \qquad \sigma_{T_3} = (1) + (1,3) - (1,2) - (1,2,3)$$

$$\varepsilon_{T_4} = (1) - (1,2) - (1,3) - (2,3) + (1,2,3) + (1,3,2) = \sigma_{T_4}$$

Higher Specht Polynomials:

Specht Module	Young Tableaux	Basis over F_T^V	Basis over H_T^V
V_{triv}	(T_1, T_1)	1	1
$V_{(2,1)}$	(T_2, T_2)	$x_3 - x_1$	$2x_3 - x_1 - x_2$
	(T_3, T_2)	$x_2 - x_1$	$2x_2 - x_1 - x_3$
	(T_2, T_3)	$x_2(x_3 - x_1)$	$x_2x_3 + x_1x_2 - 2x_1x_3$
	(T_3, T_3)	$x_3(x_2 - x_1)$	$x_2x_3 + x_1x_3 - 2x_1x_2$
V_{det}	(T_4, T_4)	$(x_1 - x_2)(x_1 - x_3)(x_2 - x_3)$	$(x_1 - x_2)(x_1 - x_3)(x_2 - x_3)$

$$\left(\begin{bmatrix} \frac{\langle F_T^{T_1}, zH_T^{T_1}\rangle}{\langle F_T^{T_1}, F_{T'}^{T'_1}\rangle} & \cdots & \frac{\langle F_T^{T_1}, zH_T^{T_k}\rangle}{\langle F_T^{T_1}, F_{T'}^{T'_1}\rangle} \\ \vdots & \ddots & \vdots \\ \frac{\langle F_T^{T_k}, zH_T^{T_1}\rangle}{\langle F_T^{T_k}, F_{T'}^{T'_k}\rangle} & \cdots & \frac{\langle F_T^{T_k}, zH_T^{T_k}\rangle}{\langle F_T^{T_k}, F_{T'}^{T'_k}\rangle} \end{bmatrix}, \begin{bmatrix} \frac{\langle H_{T'}^{T'_1}, zF_{T'}^{T'_1}\rangle}{\langle H_{T'}^{T'_1}, H_T^{T_k}\rangle} & \cdots & \frac{\langle H_{T'}^{T'_1}, zF_{T'}^{T'_k}\rangle}{\langle H_{T'}^{T'_1}, H_T^{T_1}\rangle} \\ \vdots & \ddots & \vdots \\ \frac{\langle H_{T'}^{T'_k}, zF_{T'}^{T'_1}\rangle}{\langle H_{T'}^{T'_k}, H_T^{T_k}\rangle} & \cdots & \frac{\langle H_{T'}^{T'_k}, zF_{T'}^{T'_k}\rangle}{\langle H_{T'}^{T'_k}, H_T^{T_k}\rangle} \end{bmatrix} \right)$$

S_4 matrices

$$\begin{pmatrix} \frac{1}{48} f_1^2 f_3^2 - \frac{1}{16} f_1^2 f_2 f_4 - \frac{1}{16} f_2 f_3^2 + \frac{1}{6} f_2^2 f_4 + \frac{1}{12} f_1 f_3 f_4 - \frac{2}{3} f_4^2 & \frac{1}{192} f_1 f_2 f_3^2 - \frac{1}{48} f_1 f_2^2 f_4 + \frac{1}{64} f_1^2 f_3 f_4 - \frac{3}{64} f_3^3 + \frac{1}{6} f_2 f_3 f_4 - \frac{1}{4} f_1 f_4^2 & \frac{1}{96} f_2^2 f_3^2 - \frac{1}{32} f_1 f_3^3 - \frac{1}{24} f_2^3 f_4 + \frac{7}{48} f_1 f_2 f_3 f_4 - \frac{3}{16} f_1^2 f_4^2 - \frac{1}{16} f_3^2 f_4 + \frac{1}{6} f_2 f_4^2 \\ -\frac{1}{32} f_1^2 f_2 f_3 + \frac{9}{32} f_1^3 f_4 + \frac{1}{8} f_2^2 f_3 - \frac{3}{32} f_1 f_3^2 - f_1 f_2 f_4 + \frac{3}{2} f_3 f_4 & -\frac{1}{32} f_1^2 f_3^2 + \frac{3}{32} f_1^3 f_2 f_4 + \frac{3}{32} f_2 f_3^2 - \frac{1}{4} f_1^2 f_4 - \frac{1}{8} f_1 f_3 f_4 + f_4^2 & -\frac{1}{64} f_1 f_2 f_3^2 + \frac{1}{16} f_1 f_2^2 f_4 - \frac{3}{64} f_1^2 f_3 f_4 + \frac{9}{64} f_3^3 - \frac{1}{2} f_2 f_3 f_4 + \frac{3}{4} f_1 f_4^2 \\ \frac{1}{16} f_1^2 f_2^2 - \frac{3}{16} f_1^3 f_3 - \frac{1}{4} f_2^3 + \frac{7}{8} f_1 f_2 f_3 - \frac{3}{8} f_1^2 f_4 - \frac{9}{8} f_3^2 + f_2 f_4 & \frac{1}{64} f_1^2 f_2 f_3 - \frac{9}{64} f_1^3 f_4 - \frac{1}{16} f_2^2 f_3 + \frac{3}{64} f_1 f_3^2 + \frac{1}{2} f_1 f_2 f_4 - \frac{3}{4} f_3 f_4 & -\frac{1}{32} f_1^2 f_3^2 - \frac{3}{32} f_1^3 f_2 f_4 - \frac{3}{32} f_2 f_3^2 + \frac{1}{4} f_2^2 f_4 + \frac{1}{8} f_1 f_3 f_4 - f_4^2 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{1}{36} f_1 f_2 f_3 + \frac{1}{4} f_1^2 f_4 + \frac{1}{4} f_3^2 - \frac{8}{9} f_2 f_4 & -\frac{1}{72} f_1^2 f_3^2 + \frac{1}{24} f_1^2 f_2 f_4 + \frac{1}{24} f_2 f_3^2 - \frac{1}{6} f_2^2 f_4 + \frac{1}{9} f_1 f_3 f_4 - \frac{2}{9} f_4^2 \\ \frac{2}{9} f_2^2 - \frac{2}{3} f_1 f_3 + \frac{8}{3} f_4 & \frac{1}{36} f_1 f_2 f_3 - \frac{1}{4} f_1^2 f_4 - \frac{1}{4} f_3^2 + \frac{8}{9} f_2 f_4 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{80} f_1 f_2^2 f_3 - \frac{1}{30} f_1^2 f_2^2 - \frac{1}{20} f_1^2 f_2 f_4 + \frac{2}{15} f_1^2 f_3^2 - \frac{1}{8} f_2 f_3 f_4 + \frac{5}{16} f_1 f_4^2 \\ \frac{1}{20} f_2^2 f_3 - \frac{1}{24} f_1 f_2^2 + \frac{1}{16} f_1 f_2 f_4 - \frac{25}{16} f_4^2 \\ -\frac{1}{40} f_1^2 f_2 f_3 + \frac{2}{15} f_1^3 f_4 - \frac{3}{20} f_2 f_3^2 + \frac{9}{16} f_2^2 f_4 - \frac{1}{4} f_1 f_3 f_4 \\ -\frac{3}{10} f_2^3 + \frac{1}{15} f_1 f_2 f_3 - \frac{3}{8} f_1^2 f_4 + \frac{2}{3} f_3 f_4 \\ \frac{1}{10} f_1 f_2^2 - \frac{1}{16} f_1^2 f_3 + \frac{3}{8} f_3^2 - \frac{5}{2} f_2 f_4 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{320} f_2^3 f_3 - \frac{1}{30} f_1 f_2 f_3^2 + \frac{1}{360} f_1 f_2^2 f_4 + \frac{3}{20} f_1^2 f_3 f_4 + \frac{3}{20} f_1^2 f_4^2 - \frac{1}{12} f_2^2 f_4 - \frac{5}{64} f_2 f_3^2 \\ -\frac{1}{120} f_1^2 f_2 f_3 + \frac{3}{20} f_1^3 f_4 - \frac{1}{20} f_2 f_3^2 + \frac{1}{10} f_2^2 f_4 - \frac{3}{2} f_1 f_3 f_4 \\ -\frac{3}{320} f_1 f_2^2 f_3 + \frac{1}{20} f_1^2 f_3^2 + \frac{3}{160} f_1^2 f_2 f_4 - \frac{1}{10} f_2^3 + \frac{3}{8} f_2 f_3 f_4 - \frac{15}{64} f_1 f_4^2 \\ \frac{1}{120} f_1^2 f_2^2 - \frac{1}{40} f_1^3 f_3 - \frac{1}{80} f_2^2 f_3 + \frac{1}{10} f_1 f_3^2 - \frac{11}{32} f_1 f_2 f_4 + \frac{27}{16} f_4^2 \\ \frac{1}{40} f_2^2 - \frac{1}{15} f_1 f_2 f_3 + \frac{3}{8} f_1^2 f_4 - \frac{5}{6} f_3 f_4 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{480} f_1^2 f_2 f_3^2 - \frac{1}{480} f_1^3 f_2^2 f_4 - \frac{1}{80} f_1^2 f_3 f_4 + \frac{1}{10} f_2 f_3^3 - \frac{131}{1440} f_2^2 f_3 f_4 + \frac{1}{20} f_1 f_2^3 f_4 + \frac{19}{384} f_1 f_2 f_4^2 + \frac{25}{384} f_4^3 \\ \frac{1}{20} f_1 f_2^2 f_3 - \frac{1}{160} f_1^2 f_3 f_4 + \frac{1}{4} f_2^2 f_4 - \frac{15}{32} f_3 f_4^2 \\ -\frac{1}{160} f_1^3 f_2^2 + \frac{3}{160} f_1^2 f_2 f_4 - \frac{9}{320} f_1^2 f_3^2 + \frac{9}{40} f_1 f_2^2 f_4 + \frac{9}{20} f_1^2 f_4^2 - \frac{633}{5120} f_1 f_2 f_3 f_4 + \frac{3}{32} f_1^2 f_3^2 - \frac{5}{64} f_3 f_4^2 \\ -\frac{3}{320} f_1 f_2^2 f_3 + \frac{1}{20} f_1^2 f_3^2 + \frac{1}{320} f_1^2 f_2 f_4 - \frac{1}{10} f_2^3 + \frac{3}{8} f_2 f_3 f_4 - \frac{15}{64} f_1 f_4^2 \\ \frac{1}{80} f_1^2 f_2 f_3 - \frac{3}{80} f_1^3 f_4 + \frac{1}{20} f_2 f_3^2 - \frac{9}{16} f_2^2 f_4 + \frac{1}{4} f_1 f_3 f_4 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{320} f_2^3 f_4 - \frac{1}{30} f_1 f_2 f_3 f_4 + \frac{3}{40} f_1^2 f_4^2 - \frac{3}{16} f_2 f_4^2 \\ -\frac{1}{240} f_1^2 f_2^2 + \frac{1}{20} f_1^2 f_3 f_4 - \frac{3}{20} f_3^2 + \frac{3}{16} f_2 f_3 f_4 - \frac{5}{16} f_1 f_4^2 \\ -\frac{1}{320} f_1 f_2^2 f_4 + \frac{1}{320} f_1^2 f_3 f_4 - \frac{3}{16} f_1^2 f_4^2 + \frac{15}{64} f_2 f_4^2 \\ \frac{1}{480} f_1^2 f_2 f_3 - \frac{1}{160} f_1^3 f_4 + \frac{1}{10} f_2 f_3^2 - \frac{3}{16} f_2^2 f_4 + \frac{1}{4} f_1 f_3 f_4 \\ \frac{3}{160} f_2^2 f_3 - \frac{1}{15} f_1 f_3^2 + \frac{1}{32} f_1 f_2 f_4 - \frac{99}{32} f_4^2 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{160} f_1 f_2^2 f_4 - \frac{11}{480} f_1^2 f_2 f_3 f_4 + \frac{3}{80} f_1^3 f_4^2 - \frac{1}{120} f_2 f_3^2 f_4 + \frac{1}{20} f_2^2 f_4^2 - \frac{1}{32} f_3 f_4 f_4^2 \\ \frac{3}{160} f_2^2 f_3^2 - \frac{1}{15} f_1 f_3^3 - \frac{9}{160} f_2^3 f_4 + \frac{57}{160} f_1 f_2 f_3 f_4 - \frac{25}{32} f_1^2 f_4^2 - \frac{57}{32} f_3 f_4^2 \\ -\frac{1}{160} f_1^2 f_2^2 f_4 + \frac{1}{120} f_1^3 f_3 f_4 - \frac{1}{160} f_2^2 f_3 f_4 + \frac{1}{20} f_1 f_2^2 f_4 + \frac{11}{256} f_1 f_2 f_4^2 + \frac{45}{256} f_4^3 \\ -\frac{3}{320} f_2^3 f_3 + \frac{1}{30} f_1 f_2 f_3^2 + \frac{1}{320} f_1 f_2^2 f_4 - \frac{1}{10} f_1^2 f_3 f_4 + \frac{1}{8} f_3^2 f_4 - \frac{15}{64} f_2 f_4^2 \\ \frac{1}{80} f_1 f_2^2 f_3 - \frac{1}{20} f_1^2 f_3^2 + \frac{1}{80} f_1^2 f_2 f_4 - \frac{1}{15} f_3^3 + \frac{1}{4} f_2 f_3 f_4 - \frac{15}{16} f_1 f_4^2 \end{pmatrix}$$