

### Tip sheet for the final exam

The final is based on the material covered throughout the term. You will be asked to state and prove one of the following theorems:

1. A graph is bipartite if and only if it has no cycles of odd length.
2. Every walk from  $u$  to  $v$  contains a path from  $u$  to  $v$ . Every closed trail has a subwalk that is a cycle.
3. Eulerian graph characterization and the lemma before it (lemma: a graph with minimum degree 2 contains a cycle).
4. The theorem on the characterization of trees (5 equivalent definitions of trees).
5. The correctness of Dijkstra's algorithm.
6. A DFS tree has no cross-edge.
7. If  $G$  is a chromatically  $k$ -critical graph then no vertex of  $G$  has degree less than  $k - 1$ .
8. Brook's Theorem.
9. The five-color theorem.
10. König's Theorem: If  $G$  is bipartite then  $\chi'(G) = \delta_{\max}(G)$ .

**Note:** The bonus questions of the assignments are not included in the final exam.