Math 3330 - Assignment 1 - Fall 2011.
Deadline: Thursday, September 22nd, 2:35 pm.
Late assignments (5 minutes to three days) will be penalized 10%.

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1. Let $n \geq 2$, and G be a simple graph with n vertices. Prove that there are vertices v and w of G such that $\deg(v) = \deg(w)$.

2. Without using the recursive method of Theorem 3 of Lecture 3 and the corollary thereafter, determine whether the following sequences are graphic or not:

   (i) $\langle 4, 2, 2, 1, 0, 0 \rangle$.
   (ii) $\langle 2, 2, 2, 2 \rangle$.
   (iii) $\langle 4, 3, 2, 1, 0 \rangle$.
   (iv) $\langle 4, 4, 4, 4, 3, 3, 3, 3 \rangle$.
   (v) $\langle 3, 2, 2, 1, 0 \rangle$.

3. Using the recursive method of Theorem 3 of Lecture 3 and the corollary thereafter, determine whether the following sequences are graphic or not:

   (i) $\langle 7, 7, 6, 5, 4, 4, 3, 2 \rangle$.
   (ii) $\langle 4, 4, 3, 3, 3, 3, 2, 2, 2 \rangle$.
   (iii) $\langle 5, 5, 4, 3, 2, 2, 2, 1 \rangle$.
   (iv) $\langle 5, 5, 4, 4, 2, 2, 1, 1 \rangle$.

4. For each of the following sequences list all the graphs that realize the sequence:

   (i) $\langle 10, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$.
   (ii) $\langle 2, 2, 2, 2, 2, 2, 2, 1, 1 \rangle$.
   (iii) $\langle 4, 4, 4, 4, 4 \rangle$.

5. Prove or disprove: There exists a simple graph G with 13 vertices, 31 edges, three vertices of degree one, and seven vertices of degree four.

Comments: The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.