Math 3330 - Assignment 2 - Fall 2011.

Deadline: Thursday, September 29th, 2:35 pm.

Late assignments (5 minutes to three days) will be penalized 10%.

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1. Provide a simple graph as an example for each of the following cases. Provide an explanation in each case.
   (i) A walk that is not a trail.
   (ii) A trail that is not a path.
   (iii) A closed trail that is not a cycle.
   (iv) A nontrivial closed walk that does not contain any cycles.

2. Let $k \geq 1$ be an integer. Let G be a simple graph whose vertices all have degree at least k. Prove that
   (i) G contains a path of length k.
   (ii) If $k \geq 2$ then G contains a cycle of length at least k.

3. Let $n \geq 3$ be an integer. We define the complete bipartite graph $K_{n,n}$ on the vertex set
$$V_{K_{n,n}} = \{v_1, \ldots, v_n\} \cup \{u_1, \ldots, u_n\},$$
where $X = \{v_1, \ldots, v_n\}$ and $Y = \{u_1, \ldots, u_n\}$ form a bipartite partition for $K_{n,n}$. Moreover, for every $1 \leq i, j \leq n$, v_i is adjacent to u_j. Let x and y be two different nonadjacent vertices in $K_{n,n}$.
   (i) Find the number of all the paths from x to y of length 2.
   (ii) Find the number of all the paths from x to y of length 3.
   (iii) Find the number of all the paths from x to y of length 4.
   (iv) In general, for every positive integer k, find the number of paths from x to y in $K_{n,n}$ of length k.

4. Let P and Q be paths of maximum length in a connected simple graph G. Prove that P and Q have a common vertex.

5. Prove that every closed walk W of odd length in a simple graph contains a cycle.
   Hint: First show that if W does not contain any cycles then there exists an edge in W which repeats immediately.

Comments: The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.