- 1. Prove or disprove each of the following statements:
  - (i) Every graph with fewer edges than vertices has a component that is a tree.
  - (ii) If a simple graph G has no cut-edge then every vertex of G has even degree.
  - (iii) A graph is a forest if and only if every connected subgraph is an induced subgraph.
- 2. Suppose the average degree of the vertices of a connected graph is exactly 2. How many cycles does G have? Support your answer.
- 3. Prove that a graph G is a forest if and only if every induced subgraph of G contains a vertex of degree 0 or 1.
- 4. Let  $d_1 \ge d_2 \ge \ldots \ge d_n$  be positive integers with  $n \ge 2$ . Prove that there exists a tree with vertex degrees  $d_1, \ldots, d_n$  if and only if  $\sum_{i=1}^n d_i = 2n 2$ .
- 5. Let T be a tree in which all vertices adjacent to leaves have degree at least 3. Prove that there exists a pair  $(v_1, v_2)$  of leaves that have a common neighbor.

**Comments:** The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.