Math 3330 - Assignment 5 - Fall 2011.

Deadline: Tuesday, November 29th, 2:35 pm.

Late assignments (5 minutes to three days) will be penalized 10%.

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1. Prove that every graph G has a vertex ordering relative to which the sequential vertex-coloring algorithm uses $\chi(G)$ colors.
2. For every $k \in \mathbb{N}$, construct a tree T_k with maximum degree k and an ordering $v_1, v_2, \ldots, v_n$ of its vertices with respect to which the sequential vertex-coloring algorithm uses $k+1$ colors.
3. Prove that
   1. Adding an edge to a graph increases the chromatic number by at most 1.
   2. Deleting a vertex from a graph decreases the chromatic number by at most 1.
4. Describe how to construct a connected graph with chromatic number c and independence number a, for arbitrary $a \geq 1$ and $c \geq 2$.
5. Prove that every k-chromatic graph has at least $\frac{k(k-1)}{2}$ edges.
6. Let G be a graph. We define the complement of G, denoted by $\overline{G}$, as follows:

   $$V_{\overline{G}} = V_G, \quad \forall x, y \in V_G,\ x \text{ is adjacent to } y \text{ in } G \Leftrightarrow x \text{ is not adjacent to } y \text{ in } \overline{G}.$$

   Construct a graph G with a vertex v such that $\chi(G-v) < \chi(G)$ and $\chi(\overline{G}-v) < \chi(\overline{G})$.
7. **Bonus:** Prove that if $\overline{G}$ is bipartite, then $\chi(G) = \omega(G)$, where $\omega(G)$ denotes the size of the largest clique in G.

Comments: The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.