Math 3330 - Assignment 6 - Fall 2011.
Deadline: Monday, December 12th.
Late assignments will be penalized 10%.

---

1. In each case, either prove the statement or disprove it by providing a counterexample.
   (i) Subdividing an edge e in a graph G causes the edge-chromatic number to increase by at most 1.
   (ii) Subdividing an edge e in a graph G causes the edge-chromatic number to decrease by at most 1.
2. In each of the following cases, determine if the graph is planar or not. If the graph is planar, draw its embedding with no edge-crossings. If not, provide a proof.
   (i) Peterson graph.
   (ii) $K_{3,3} - e$, where e is any edge in the complete bipartite graph $K_{3,3}$.
   (iii) $K_8 - K_{3,3}$, where K_8 is a complete graph on 8 vertices.
   (iv) $K_{6,6} - C_{12}$, where $k_{6,6}$ is a complete bipartite graph with partitions of size 6, and C_{12} is a cycle of length 12.
3. Let G be a minimal non-planar graph, i.e. G is non-planar, but every subgraph of G is planar.
   (i) Prove that G is connected.
   (ii) Prove that G does not have any cut-vertices.
4. Prove that every planar graph G decomposes into two bipartite graphs. I.e. there are bipartite subgraphs G_1 and G_2 of G such that
$$E_{G_1} \cap E_{G_2} = \emptyset \text{ and } E_G = E_{G_1} \cup E_{G_2}.$$
5. Give an explicit edge-coloring to prove that $\chi'(K_{r,s}) = \delta_{\max}(K_{r,s})$.
6. Prove that for any tree T we have $\chi'(T) = \delta_{\max}(T)$.
7. **bonus:** Let G be a regular graph with a cut vertex. Prove that $\chi'(G) > \delta_{\max}(G)$.
   Remark: The bonus question is worth 20 % , and is due on December 17th.

Comments: The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.