Math 4020/5020 - Assignment 1 - Winter 2012.

Deadline: Wednesday, January 18th 11:35 am.

Late assignments (5 minutes to two days) will be penalized 10%.

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1. Let w be a nonzero complex number, and n be a positive integer. Present a complete list of n'th roots of w. Prove the following three facts about your list:
   (i) Every element of your list is an n'th root of w.
   (ii) Each element of your list is distinct.
   (iii) Your list is complete, i.e. it contains all the n'th roots of w.

2. Let $\rho > 1$, and $z_0, z_1 \in \mathbb{C}$ be fixed. Prove that the set of all the complex points z that satisfy the equation

$$|z - z_0| = \rho |z - z_1|$$

   forms a circle in the complex plane. Find the center and radius of that cycle.

3. Let D be a proper subset of $\mathbb{C}$. Prove that
   (i) The set of boundary points of D is the same as the set of boundary points of $\mathbb{C} \setminus D$.
   (ii) D is open if and only if D has no boundary points.
   (iii) D is closed if and only if it includes all its boundary points.

4. Let f be a complex function whose real and imaginary components are u and v, i.e. $f(z) = u(x, y) + iv(x, y)$ where $z = x + iy$. Let $z_0 = x_0 + iy_0$ and $w_0 = s_0 + it_0$ be two complex numbers. Prove that if $\lim_{x \to x_0, y \to y_0} u(x, y) = s_0$ and $\lim_{x \to x_0, y \to y_0} v(x, y) = t_0$ then $\lim_{z \to z_0} f(z) = w_0$.

5. Suppose that f and g are complex functions, and M and L are complex numbers such that $\lim_{z \to z_0} f(z) = M$ and $\lim_{z \to z_0} g(z) = L$. Prove that $\lim_{z \to z_0} (fg)(z) = ML$.

Comments:s The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.