

Nova Scotia

Math League

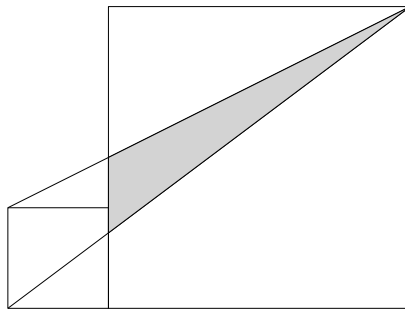
2014–2015

Game Three

PROBLEMS

Team Questions

1. Find the 2015th digit after the decimal point when $\frac{1}{7}$ is written in decimal form.
2. The parabolas $y = x^2 - 3x + 2$ and $y = 3x^2 - 8x - 1$ intersect in two points. Find the x -intercept of the line passing through these points.
3. The figure below shows two adjacent squares, with lines drawn from two corners of the small square to a corner of the large square. The small square has area 16 and the large square has area 144. Find the area of the shaded triangle.



4. A rectangular field measures 300m by 600m. A fence is erected around the field, with posts spaced 10m apart along the perimeter (including one post in each corner).
Unfortunately, the posts *should* have been installed 8m apart on the long sides of the field and 15m apart on the short sides.
What is the minimum number of posts that will have to be moved to correct this error?

5. Bob throws a fair 6-sided die three times in a row. Find the probability that his highest roll is a 5.

6. How many integers between 1 and 30 (inclusive) can be expressed as a sum of two or more consecutive positive integers?

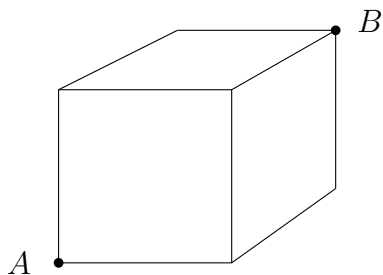
For instance, 12 is such a number since $12 = 3 + 4 + 5$.

7. When 1 litre of water is poured into a circular cone it reaches a depth of 5cm. What is the depth when an additional litre of water is poured into the cone?

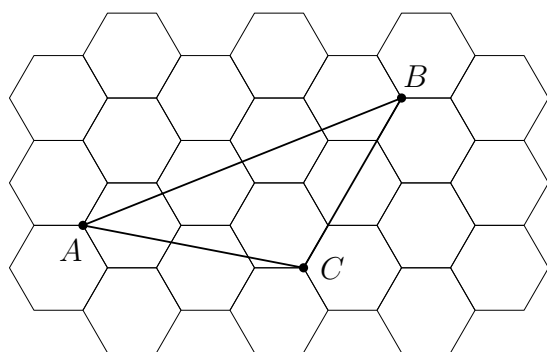
8. Evaluate the following product:

$$\frac{4}{3} \cdot \frac{9}{8} \cdot \frac{16}{15} \cdot \frac{25}{24} \cdots \frac{2015^2}{2015^2 - 1}$$

9. An ant walks from corner A of a cube to the diagonally opposite corner B . If the ant walks only along edges and never hits the same point twice, how many distinct paths can it take?



10. The side of each regular hexagon in the honeycomb lattice below is of length 1. Find the perimeter of $\triangle ABC$.



Pairs Relay

P-A. Let $A = \left(\sqrt{2 + \sqrt{3}} + \sqrt{2 - \sqrt{3}} \right)^2$.

Pass on A

P-B. You will receive A.

The lines $y = Ax + B$, $y = 2Ax + 2$ and $y = 2x + 2A$ all pass through the same point.

Pass on B

P-C. You will receive B.

Real numbers x and y satisfy $\frac{3x + 2y}{2x - 3y} = B$.

Let $C = \frac{x^2}{y^2}$.

Pass on C

P-D. You will receive C.

A party is attended by C couples. Each person shakes hands with everyone except their partner (and themselves!)

Let D be the total number of handshakes.

Done!

Individual Relay

I-A. Reducing two sides of an $A \times A$ square by 60% and enlarging the other two by 60% results in a rectangle with an area of 64 square units.

Pass on A

I-B. You will receive A.

Let B be the smallest positive integer such that $20B + A$ is divisible by 7.

Pass on B

I-C. You will receive B.

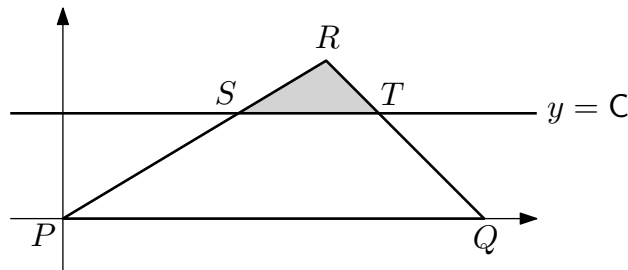
For any nonzero x and y , let $x \oplus y = \frac{x + y}{xy}$.

Compute $C = (B \oplus 1) \oplus (1 \oplus B)$.

Pass on C

I-D. You will receive C.

Triangle PQR has $P = (0,0)$, $Q = (8,0)$ and $R = (5,3)$. The line $y = C$ intersects PR and QR at S and T , respectively.



Let D be the area of $\triangle RST$.

Done!