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Three new extraction formulae,
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Abstract

Let α be an irrational number between 0 and 1. Let a and b be distinct letters. Define $d_n = a$ (resp., b) if $[(n+1)\alpha] - [n\alpha] = 0$ (resp., 1), $n \in \mathbb{Z}$. Define x to be the two-way infinite word whose n^{th} letter is $d_n, n \in \mathbb{Z}$. Define $x_m = d_{m+1}d_{m+2}\cdots, m \in \mathbb{Z}$, $s_0 = \varepsilon$, the empty word, $s_m = d_1 d_2 \cdots d_m, m \geq 1$. The problem of determining the extracted word $\langle x_m, x_0\rangle$ obtained by aligning x_m with x_0 was originally posed by D.R. Hofstadter in 1963. Known extraction formulae include $\langle x_m, x_0\rangle$ $(m > 0)$ (by R.J. Hendel and S.A. Monteferrante 1994), $\langle x_0, x_m\rangle$ $(m \geq 1)$ (by W. Chuan 1995) for $\alpha = (\sqrt{5}-1)/2$ and partial results for $\langle x_m, x_0\rangle$ $(m \geq 1)$ (by R.J. Hendel 1996) and all cases of $\langle x_0, x_m\rangle$ $(m \geq 0)$ (by W. Chuan and F. Yu 2000) for $\alpha = \sqrt{2}-1$. In this short note, we establish the following three new extraction formulae for $\alpha = (\sqrt{5}-1)/2$:

$$\langle x_m, x_{-2}\rangle = x_m \ (m > -2)$$
$$\langle x_m, x_{-2}\rangle = R(s_{-m-2}) \ (m \leq -2)$$
$$\langle x_0, x_{-m}\rangle = \begin{cases} x_{m-2} \ (m > 1) \\ bx_0 \neq x_{-1} \ (m = 1) \end{cases}$$

which involve x_m, where $m < 0$. We also show that the first formula is equivalent to the formula proved by Hendel and Monteferrante.