Arthur T. Benjamin, Phyllis Chinn, Jacob N. Scott, and Greg Simay
Combinatorics of Two-Toned Tilings,
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Abstract

We introduce the function $a(r, n)$ which counts tilings of length $n+r$ that utilize white tiles (whose lengths can vary between 1 and n) and r identical red squares. These tilings are called two-toned tilings. We provide combinatorial proofs of several identities satisfied by $a(r, n)$ and its generalizations, including one that produces kth order Fibonacci numbers. Applications to integer partitions are also provided.