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Abstract

We study the following alternating sums,

$$f_n(m) \equiv \sum_{k \geq 0} (-1)^k \binom{n-k}{k} C_{n+m-k}, \quad n \geq 0, \quad m \geq -1$$

where C_n is the nth Catalan number, and we express the results as closed forms that can be represented as polynomials of degree m in n. We show that the number functions $f_n(m)$ are: 1) integral-valued; 2) positive-definite in sign; and 3) have a common factor, $n+1$, for $n \geq 0$, $m \geq 1$. We also show how to obtain the coefficients in the polynomial representation in powers of n.