Lawrence Somer and Michal Křížek
Identically Distributed Second-Order Linear Recurrences Modulo p,
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Abstract

Let $w(a,-1)$ denote the second-order linear recurrence satisfying the recursion relation

$$w_{n+2} = aw_{n+1} - w_n,$$

where a and the initial terms w_0, w_1 are all integers. Let p be an odd prime. The *restricted period* $h_w(p)$ of $w(a,-1)$ modulo p is the least positive integer r such that $w_{n+r} \equiv Mw_n \pmod{p}$ for all $n \geq 0$ and some nonzero residue M modulo p. We distinguish two recurrences, the Lucas sequence of the first kind $u(a,-1)$ and the Lucas sequence of the second kind $v(a,-1)$, satisfying the above recursion relation and having initial terms $u_0 = 0$, $u_1 = 1$ and $v_0 = 2$, $v_1 = a$, respectively. We show that if $u(a_1,-1)$ and $u(a_2,-1)$ both have the same restricted period modulo p, or equivalently, the same period modulo p, then $u(a_1,-1)$ and $u(a_2,-1)$ have the same distribution of residues modulo p. Similar results are obtained for Lucas sequences of the second kind.