Andrew Bulawa and Whan Ki Lee

Integer Values of Generating Functions for the Fibonacci and Related Sequences,

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Abstract

It is known that the generating function of the Fibonacci sequence, $F(x) = \sum F_i x^i = x + x^2 + 2x^3 + 3x^4 + 5x^5 + \cdots$, attains an integer value if $x = F_i/F_{i+1}$ for any non-negative integer i. It has been conjectured that those values constitute *all* rational numbers, in the interval of convergence of F, that result in $F(x) \in \mathbb{Z}$. In this paper we prove this conjecture. We also extend these results to the class of sequences satisfying the recursion relation $R_{i+2} = aR_{i+1} + bR_i$ with initial values $(R_0, R_1) = (0, 1)$, where a and b are positive integers satisfying $b \mid a$.