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Sommerville's Symmetrical Cyclic Compositions of a Positive Integer with Parts Avoiding Multiples of an Integer,
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Abstract

A linear composition of a positive integer N is an ordered list of positive integers (called parts) whose sum equals N. A linear composition of N is called palindromic of type I if it stays the same when it is read in reverse order, while it is called palindromic of type II if it becomes a palindromic composition of type I (of an integer smaller than N) when we remove the first part. By considering all cyclic shifts of a linear composition of N as equivalent linear compositions, we may define a cyclic composition of N. Cyclic compositions were originally studied by D. M. Y. Sommerville more than a century ago, who also considered symmetrical cyclic compositions of N. In this paper, we prove that the equivalence class of every symmetrical cyclic composition of N with length K (excluding the one with all parts equal when K divides N) contains exactly two linear palindromic compositions of type I or II. Using this result, we derive generating functions for the cardinalities of classes of symmetrical cyclic compositions of N that avoid integers in a set A. We then derive general recurrences for the cardinalities of these classes of symmetrical cyclic compositions. When A consists of all multiples of a positive integer r, we use these recurrences to derive Fibonacci-type recurrences. We also indicate that the number of dihedral compositions of N with K parts in A is the average of the corresponding numbers of cyclic compositions and Sommerville's symmetrical cyclic compositions.