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On Some Arithmetic Properties of a Sequence Related to the Quotient of Fibonacci Numbers,

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Abstract

We examine the sequence $(T_n)_{n\geq 1}$ of numbers: 1, 11, 61, 451, 3001, 20801, 141961, . . . given by $T_n = F_{5n}/(5F_n)$, where F_n is the Fibonacci number. Curious divisibility properties are obtained including related conditions resembling a strong divisibility sequence. In particular, we prove that all prime divisors of the numbers in this sequence end in one. Another result asserts that each integral power of a number in the sequence is a divisor of some other number in the sequence. Specifically, we prove that for any positive integers n and k, the term

$$T(nT(nT(\cdots nT(n)\cdots)))$$

with k occurrences of the number n is exactly divisible by T_n^k .