Christian Ballot, Clark Kimberling, and Peter J. C. Moses
Linear Recurrences Originating From Polynomial Trees,
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Abstract

Let T^* be the set of polynomials in x generated by these rules: $0 \in T^*$, and if $p \in T^*$, then $p+1 \in T^*$ and $xp \in T^*$. Let $g(0) = \{0\}$, $g(1) = \{1\}$, $g(2) = \{2, x\}$, and so on, so that the cardinality of $g(n)$ is given by $G_n = 2^{n-1}$ for $n \geq 1$, and T^* can be regarded as a tree whose nth generation consists of nodes labeled by the polynomials in $g(n)$. Let $T(r)$ be the subtree of T^* obtained by substituting r for x and deleting duplicates. For various choices of r, the cardinality sequence G_n satisfies a linear recurrence relation.