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A new combinatorial interpretation of the Fibonacci numbers squared,
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Abstract

We consider the tiling of an n -board (a $1 \times n$ array of square cells of unit width) with half-squares ($\frac{1}{2} \times 1$ tiles) and $(\frac{1}{2}, \frac{1}{2})$ -fence tiles. A $(\frac{1}{2}, \frac{1}{2})$ -fence tile is composed of two half-squares separated by a gap of width $\frac{1}{2}$. We show that the number of ways to tile an n -board using these types of tiles equals F_{n+1}^2 where F_n is the n th Fibonacci number. We use these tilings to devise combinatorial proofs of identities relating the Fibonacci numbers squared to one another and to other number sequences. Some of these identities appear to be new.