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Re³counting the Rationals,
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Abstract

In 1999, Neil Calkin and Herbert Wilf wrote “Recounting the rationals” which gave an explicit bijection between the positive integers and the positive rationals. We find several different (some new) ways to construct this enumeration and thus create pointers for generalizing. Next, we use circle packings to generalize and find two other enumerations. Surprisingly, the three enumerations are all that are possible by using this technique. The proofs involve, among other things, “negative” continued fractions, Chebyshev polynomials, Euler’s totient function, and generalizations of Stern’s diatomic sequence. Finally we look at some of the remarkable similarities – and differences – of these sequences.