Abstract

We obtain exact divisibility results for the powers of the balancing and Lucas-balancing numbers. This gives all the results analogous to those of Fibonacci and Lucas numbers from 1970 to 2019. For example, Hoggatt and Bicknell-Johnson (1977) and Benjamin and Rouse (2009) proved that if $F_n^k \mid m$, then $F_n^{k+1} \mid F_{nm}$, which was later generalized by Pongsriiam (2014) to include the exact divisibility such as $F_n^{k+1} \parallel F_{nm}$, provided that $F_n^k \parallel m$, $n \geq 3$, and $n \not\equiv 3 \pmod 6$. Here, F_n is the nth Fibonacci number. For the balancing numbers B_n, we show that $B_n^k \parallel m$ if and only if $B_n^{k+1} \parallel B_{nm}$ for all $k \geq 1$ and $m, n \geq 2$. The corresponding results for the Lucas-balancing numbers are also given.