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Fence tiling derived identities involving the metallonacci numbers squared or cubed,
Fibonacci Quart. **60** (2022), no. 5, 5–17.

Abstract We refer to the generalized Fibonacci sequence $(M_n^{(c)})_{n\geq 0}$, where $M_{n+1}^{(c)} = cM_n^{(c)} + M_{n-1}^{(c)}$ for $n > 0$ with $M_0^{(c)} = 0$, $M_1^{(c)} = 1$, for $c = 1, 2, \ldots$ as the c-metallonacci numbers. We consider the tiling of an n-board (an $n\times 1$ rectangular board) with c colours of $1/p\times 1$ tiles (with the shorter sides always aligned horizontally) and $(1/p, 1-1/p)$-fence tiles for $p \in \mathbb{Z}^+$. A (w, g)-fence tile is composed of two $w \times 1$ sub-tiles separated by a $g \times 1$ gap. The number of such tilings equals $(M_{n+1}^{(c)})^p$ and we use this result for the cases $p = 2, 3$ to devise straightforward combinatorial proofs of identities relating the metallonacci numbers squared or cubed to other combinations of metallonacci numbers. Special cases include relations between the Pell numbers cubed and the even Fibonacci numbers. Most of the identities derived here appear to be new.