Michael A. Allen and Kenneth Edwards
Identities Involving the Tribonacci Numbers Squared Via Tilings with Combs,
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Abstract

The number of ways to tile an n-board (an $n \times 1$ rectangular board) with $(\frac{1}{2}, \frac{1}{2}; 1)$-, $(\frac{1}{2}, \frac{1}{2}; 2)$-, and $(\frac{1}{2}, \frac{1}{2}; 3)$-combs is T_{n+2}^2, where T_n is the nth tribonacci number. A $(\frac{1}{2}, \frac{1}{2}; m)$-comb is a tile composed of m subtiles of dimensions $\frac{1}{2} \times 1$ (with the shorter sides always horizontal) separated by gaps of dimensions $\frac{1}{2} \times 1$. We use such tilings to obtain quick combinatorial proofs of identities relating the tribonacci numbers squared to one another, to other combinations of tribonacci numbers, and to the Fibonacci, Narayana's cows, and Padovan numbers. Most of these identities appear to be new.