Michael A. Jones and Brittany C. Ohlinger
Self-Similar Structure of $\mathcal{P}$-Positions of the Game Euclid,
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Abstract

Euclid is a Nim-type combinatorial game in which the game moves can be thought of as traversing branches of the Calkin–Wilf tree. In this paper, we show that the $\mathcal{P}$-positions of Euclid exhibit a self-similar structure within the tree, and we relate these $\mathcal{P}$-positions to the Fibonacci numbers. Additionally, we identify the game positions that require the maximum number of moves in optimal play, locate the vertices corresponding to these moves in the Calkin–Wilf tree, and relate them to the integer pairs that require the maximum number of steps in the Euclidean algorithm.