# F I B O N A C C I
# A N D R E L A T E D
# N U M B E R T H E O R E T I C
# T A B L E S


COMPILED BY

BROTHER ALFRED BROUSSEAU



PUBLISHED BY

THE FIBONACCI ASSOCIATION

SANTA CLARA UNIVERSITY, SANTA CLARA, CALIFORNIA

1972

# FIBONACCI AND RELATED NUMBER THEORETIC TABLES

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## FOREWORD

In the spring of 1970, a proposal to publish tables on Fibonacci sequences and number theory was sent to the members of the Fibonacci Association. The response to this suggestion was favorable. Later it was decided to produce the work in two parts so as to avoid undue bulk and also make some of the material available at an earlier date. This is the first half of the work which deals with Fibonacci sequences, linear recursion, and related matters.

Apart from a few exceptions the tables were not taken from previously published material. The origin of each table is found under Acknowledgments.

It is inevitable that there will be some errors in a publication of this type in spite of the care that is taken to avoid them. The compiler will be particularly grateful to any one bringing such defects to his attention. Because of the flexible manner in which the volume is being produced, the opportunity to make corrections will not be long delayed.

The compiler wishes to express particular gratitude to Marjorie Bicknell who checked over the entries of all tables.

Brother Alfred Brousseau

February 1972

# ACKNOWLEDGMENTS

1. THE FIBONACCI NUMBERS AND THEIR FACTORIZATIONS
Taken from RECURRING SEQUENCES, second edition, 1966, by Dov Jarden. Special recognition is given in these tables to the contributions of John Brillhart.

2. LUCAS NUMBERS AND THEIR FACTORIZATIONS
Same as for # 1.

3. SQUARES AND SUMS OF SQUARES OF FIBONACCI NUMBERS
Calculated by Brother Alfred Brousseau on the Olivetti-Underwood Programma 101.

4. CUBES AND SUMS OF CUBES OF FIBONACCI NUMBERS. Same as # 3.

5. FOURTH POWERS AND SUMS OF FOURTH POWERS OF FIBONACCI NUMBERS.
Same as # 3.

6. SQUARES AND SUMS OF SQUARES OF LUCAS NUMBERS. Same as # 3.

7. CUBES AND SUMS OF CUBES OF LUCAS NUMBERS. Same as #3.

8. FOURTH POWERS AND SUMS OF FOURTH POWERS OF LUCAS NUMBERS.
Same as #3.

9. FIBONACCI PRIMES
Taken from RECURRING SEQUENCES by Dov Jarden.

10. LUCAS PRIMES
Same as #9.

11. FIBONACCI ENTRY POINTS (primes)
Calculated by M. Wunderlich and published originally by the Fibonacci Association.

12. ENTRY POINTS AND PERIODS FOR THE FIBONACCI SEQUENCE(including composites)
Based on the work of M. Wunderlich. Additional calculations made by Brother Alfred Brousseau.

13. CHARACTERISTIC NUMBERS OF FIBONACCI SEQUENCES
Calculations made by Brother Alfred Brousseau on the Programma 101.

14. SUMS OF FIBONACCI AND LUCAS RECIPROCALS
Calculations made by Don Patterson, student at San Jose State College.

15. RESIDUE CYCLES OF FIBONACCI SEQUENCES
Calculations made by Brother Alfred Brousseau with the aid of the Programma 101.

16. ASYMPTOTIC RATIOS OF TERMS OF LINEAR RECURSION RELATIONS
Calculations made by Brother Alfred Brousseau on the Programma 101.

17. FACTORIZATIONS OF THE TERMS OF THE FIBONACCI SEQUENCE WITH $T_1 = 2$, $T_2 = 5$.
Calculations made by Ron Gurich, student at San Jose State College.

18. FACTORIZATIONS OF THE TERMS OF THE FIBONACCI SEQUENCE WITH $T_1 = 1$, $T_2 = 4$.
Same as # 17.

19. THE NUMBER OF REPRESENTATIONS S(n) OF INTEGERS AS SUMS OF DISTINCT ELEMENTS OF THE ABBREVIATED FIBONACCI SEQUENCE
Calculations made by Larry Snyder, student at San Jose State College.

20. THE NUMBER OF REPRESENTATIONS Q(n) OF INTEGERS AS SUMS OF THE LUCAS SEQUENCE
Same as # 19.

21. LIST OF INTEGERS NOT REPRESENTABLE BY THE TRUNCATED FIBONACCI SEQUENCE 2,3,5,8,....
Calculations made by Terry Brennan, Dan Morris and Scott Frye, students at San Jose State College.

22. INTEGERS NOT REPRESENTABLE BY THE TRUNCATED TRIBONACCI SEQUENCE
Same as # 21.

23. INTEGERS NOT REPRESENTABLE BY THE TRUNCATED QUADRANACCI SEQUENCE
Calculations made by Don Patterson, student at San Jose State College.

24. INTEGERS NOT REPRESENTABLE BY THE SEQUENCE 1,3,5,11,.....with $U_{n+2} = U_{n+1} + 2U_n$
Calculations made by Don Patterson, student at San Jose State College.

25. FIBONACCI FACTORIALS
Tables produced by Brother Alfred Brousseau on the basis of Fibonacci factorizations.

26. FIBONOMIAL COEFFICIENTS
Calculations made by Brother Alfred Brousseau with the aid of the Programma 101.

27. TABLES OF FIBONOMIAL COEFFICIENTS
Same as # 26.

28. FIBONACCI POLYNOMIAL COEFFICIENTS
Same as # 26.

29. CONTINUED FRACTION QUOTIENTS OF LINEAR FIBONACCI RATIOS
Formulas and calculations the work of Brother Alfred Brousseau.

30. CONTINUED FRACTION EXPANSIONS OF QUADRATIC FIBONACCI AND LUCAS RATIOS
Same as # 29.

## ACKNOWLEDGMENTS

31. CONTINUED FRACTION EXPANSION OF MULTIPLES OF THE GOLDEN SECTION RATIO
Calculations made by Brother Alfred Brousseau on the Programma 101 and checked in part against the work of C.T. Long and J.H. Jordan.

32. POWERS OF THE GOLDEN SECTION RATIO
Calculations made by Brother Alfred Brousseau on the Programma 101.

33. BINOMIAL COEFFICIENTS
Same as # 32.

34. DIAGONAL SUMS OF PASCAL'S TRIANGLE
Tables produced by Dan Morris, Scott Frye, Joe Rose, Larry Snyder, students at San Jose State College and by Marcia Silver for R.G. Tobey at Brandeis University in an experimental program.

35. SPECIAL DIAGONAL SUMS OF PASCAL'S TRIANGLE
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36. TERMS OF THE SEQUENCE WITH $T_0 = 1$, $T_1 = 1$ and $T_{n+1} = 3T_n + T_{n-1}$
Calculated by Brother Alfred Brousseau on the 360/25 at the St. Mary's Computer Services.

37. TERMS OF THE SEQUENCE WITH $T_0 = 2$, $T_1 = 3$ and $T_{n+1} = 3T_n + T_{n-1}$
Same as # 36.

38. ENTRY POINTS IN THE (3,1) SEQUENCE
Calculated by Brother Alfred Brousseau on the 360/25 at the St. Mary's Computer Services.

39. FACTOR FORMS OF PRIME FACTORS FOR THE (3,1) SEQUENCES
Formulas developed by Brother Alfred Brousseau.

40. CHARACTERISTIC NUMBERS OF (3,1) SEQUENCES
Calculations made by Brother Alfred Brousseau on the Programma 101.

41. TABLE OF COEFFICIENTS FOR THE EXPANSION OF $Y_{nk}$ IN TERMS OF $Y_k$ AND $Z_k$
Same as # 40.

42. TABLE OF COEFFICIENTS FOR THE EXPANSION OF $Z_{nk}$ IN TERMS OF $Z_k$
Same as # 40.

Note. Tables 14, 17, 18,19,20,21,22,23,24,34,and 35 were originally generated for use in Professor V.E. Hoggatt's Fibonacci research classes.

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TABLES 9 and 10. FIBONACCI AND LUCAS PRIMES

Same as for Tables 1 and 2.

TABLES 11 and 12. FIBONACCI ENTRY POINTS

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TABLE 16. ASYMPTOTIC VALUES OF TERMS OF LINEAR RECURSION RELATIONS

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TABLES 17 and 18. FACTORIZATIONS OF TERMS OF TWO CONJUGATE SEQUENCES

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TABLES 26 and 27. FIBONOMIAL COEFFICIENTS

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TABLE 31. CONTINUED FRACTION EXPANSION OF MULTIPLES OF THE GOLDEN SECTION RATIO

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TABLES 34 and 35. PASCAL TRIANGLE SEQUENCES

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TABLE 38. ENTRY POINTS OF THE (3,1) SEQUENCE

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TABLE 40. CHARACTERISTIC NUMBERS OF (3,1) SEQUENCE

1. Morgan Ward, The Characteristic Numbers of a Sequence Satisfying a Linear Recursion Relation, Trans. Amer. Math.Soc.. 33 (1931), pp. 153-165.
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TABLE 42. TABLE OF COEFFICIENTS OF THE EXPANSION OF $Z_{nk}$ IN TERMS OF $Z_k$

1. V.E.Hoggatt,Jr., An Application of the Lucas Triangle, FQ, Oct. 1970, pp. 360-364.

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THE FIBONACCI NUMBERS

In this table, primitive factors, that is, primes that divide a member of the sequence for the first time are underlined.

| n | FIBONACCI NUMBER | FACTORIZATION |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 1 |
| 3 | 2 | $\underline{2}$ |
| 4 | 3 | $\underline{3}$ |
| 5 | 5 | $\underline{5}$ |
| 6 | 8 | $2^3$ |
| 7 | 13 | $\underline{13}$ |
| 8 | 21 | $3*\underline{7}$ |
| 9 | 34 | $2*\underline{17}$ |
| 10 | 55 | $5*\underline{11}$ |
| 11 | 89 | $\underline{89}$ |
| 12 | 144 | $2^4*3^2$ |
| 13 | 233 | $\underline{233}$ |
| 14 | 377 | $13*\underline{29}$ |
| 15 | 610 | $2*5*\underline{61}$ |
| 16 | 987 | $3*7*\underline{47}$ |
| 17 | 1597 | $\underline{1597}$ |
| 18 | 2584 | $2^3*17*\underline{19}$ |
| 19 | 4181 | $\underline{37}*\underline{113}$ |
| 20 | 6765 | $3*5*11*\underline{41}$ |
| 21 | 10946 | $2*13*\underline{421}$ |
| 22 | 17711 | $89*\underline{199}$ |
| 23 | 28657 | $\underline{28657}$ |

# THE FIBONACCI NUMBERS

| n | FIBONACCI NUMBER | FACTORIZATION |
|---|---|---|
| 24 | 46368 | $2^{5}*3^{2}*7*23$ |
| 25 | 75025 | $5^{2}*3001$ |
| 26 | 121393 | 233*521 |
| 27 | 196418 | 2*17*53*109 |
| 28 | 317811 | 3*13*29*281 |
| 29 | 514229 | 514229 |
| 30 | 832040 | $2^{3}*5*11*31*61$ |
| 31 | 1346269 | 557*2417 |
| 32 | 2178309 | 3*7*47*2207 |
| 33 | 3524578 | 2*89*19801 |
| 34 | 5702887 | 1597*3571 |
| 35 | 9227465 | 5*13*141961 |
| 36 | 14930352 | $2^{4}*3^{3}*17*19*107$ |
| 37 | 24157817 | 73*149*2221 |
| 38 | 39088169 | 37*113*9349 |
| 39 | 63245986 | 2*233*135721 |
| 40 | 102334155 | 3*5*7*11*41*2161 |
| 41 | 165580141 | 2789*59369 |
| 42 | 267914296 | $2^{3}*13*29*211*421$ |
| 43 | 433494437 | 433494437 |
| 44 | 701408733 | 3*43*89*199*307 |
| 45 | 1134903170 | 2*5*17*61*109441 |
| 46 | 1836311903 | 139*461*28657 |
| 47 | 2971215073 | 2971215073 |

THE FIBONACCI NUMBERS

| n | FIBONACCI NUMBER | FACTORIZATION |
|---|---|---|
| 48 | 4807526976 | $2^6*3^2*7*23*47*1103$ |
| 49 | 7778742049 | 13*97*6168709 |
| 50 | 1258626902 5 | $5^2*11*101*151*3001$ |
| 51 | 2036501107 4 | 2*1597*6376021 |
| 52 | 3295128009 9 | 3*233*521*90481 |
| 53 | 5331629117 3 | 953*55945741 |
| 54 | 8626757127 2 | $2^3*17*19*53*109*5779$ |
| 55 | 1395838624 45 | 5*89*661*474541 |
| 56 | 2258514337 17 | $3*7^2*13*29*281*14503$ |
| 57 | 3654352961 62 | 2*37*113*797*54833 |
| 58 | 5912867298 79 | 59*19489*514229 |
| 59 | 9567220260 41 | 353*2710260697 |
| 60 | 1548008755 920 | $2^4*3^2*5*11*31*41*61*2521$ |
| 61 | 2504730781 961 | 4513*555003497 |
| 62 | 4052739537 881 | 557*2417*3010349 |
| 63 | 6557470319 842 | 2*13*17*421*35239681 |
| 64 | 1061020985 7723 | 3*7*47*1087*2207*4481 |
| 65 | 1716768017 7565 | 5*233*14736206161 |
| 66 | 2777789003 5288 | $2^3*89*199*9901*19801$ |
| 67 | 4494557021 2853 | 269*116849*1429913 |
| 68 | 7272346024 8141 | 3*67*1597*3571*63443 |
| 69 | 1176690304 60994 | 2*137*829*18077*28657 |
| 70 | 1903924907 09135 | 5*11*13*29*71*911*141961 |
| 71 | 3080615211 70129 | 6673*46165371073 |

THE FIBONACCI NUMBERS

| n | FIBONACCI NUMBER | FACTORIZATION |
|---|---|---|
| 72 | 4984540118 79264 | $2^{5}*3^{3}*7*17*19*23*107*103681$ |
| 73 | 8065155330 49393 | 9375829*86020717 |
| 74 | 1304969544 928657 | 73*149*2221*54018521 |
| 75 | 2111485077 978050 | $2*5^{2}*61*3001*230686501$ |
| 76 | 3416454622 906707 | 3*37*113*9349*29134601 |
| 77 | 5527939700 884757 | 13*89*988681*4832521 |
| 78 | 8944394323 791464 | $2^{3}*79*233*521*859*135721$ |
| 79 | 1447233402 4676221 | 157*92180471494753 |
| 80 | 2341672834 8467685 | 3*5*7*11*41*47*1601*2161*3041 |
| 81 | 3788906237 3143906 | 2*17*53*109*2269*4373*19441 |
| 82 | 6130579072 1611591 | 2789*59369*370248451 |
| 83 | 9919485309 4755497 | 99194853094755497 |
| 84 | 1605006438 16367088 | $2^{4}*3^{2}*13*29*83*211*281*421*1427$ |
| 85 | 2596954969 11122585 | 5*1597*9521*3415914041 |
| 86 | 4201961407 27489673 | 6709*144481*433494437 |
| 87 | 6798916376 38612258 | 2*173*514229*3821263937 |
| 88 | 1100087778 366101931 | 3*7*43*89*199*263*307*881*967 |
| 89 | 1779979416 004714189 | 1069*1665088321800481 |
| 90 | 2880067194 370816120 | $2^{3}*5*11*17*19*31*61*181*541*109441$ |
| 91 | 4660046610 375530309 | $13^{2}*233*741469*159607993$ |
| 92 | 7540113804 746346429 | 3*139*461*4969*28657*275449 |
| 93 | 1220016041 5121876738 | 2*557*2417*4531100550901 |
| 94 | 1974027421 9868223167 | 2971215073*6643838879 |
| 95 | 3194043463 4990099905 | 5*37*113*761*29641*67735001 |

# THE FIBONACCI NUMBERS

| n | FIBONACCI NUMBER | FACTORIZATION |
|---|---|---|
| 96 | 5168070885 4858323072 | $2^7*3^2*7*23*47*769*1103*2207*3167$ |
| 97 | 8362114348 9848422977 | 193*389*3084989*361040209 |
| 98 | 1353018523 4470674604 9 | 13*29*97*6168709*599786069 |
| 99 | 2189229958 3455516902 6 | 2*17*89*197*19801*18546805133 |
| 100 | 3542248481 7926191507 5 | $3*5^2*11*41*101*151*401*3001*570601$ |
| 101 | 5731478440 1381708410 1 | 743519377*770857978613 |
| 102 | 9273726921 9307899917 6 | $2^3*919*1597*3469*3571*6376021$ |
| 103 | 1500520536 2068960832 77 | 519121*5644193*512119709 |
| 104 | 2427893228 3999750824 53 | 3*7*103*233*521*90481*102193207 |
| 105 | 3928413764 6068711657 30 | 2*5*13*61*421*141961*8288823481 |
| 106 | 6356306993 0068462481 83 | 953*55945741*119218851371 |
| 107 | 1028472075 7613717413 913 | 1247833*8242065050061761 |
| 108 | 1664102775 0620563662 096 | $2^4*3^4*17*19*53*107*109*5779*11128427$ |
| 109 | 2692574850 8234281076 009 | 827728777*32529675488417 |
| 110 | 4356677625 8854844738 105 | $5*11^2*89*199*331*661*39161*474541$ |
| 111 | 7049252476 7089125814 114 | 2*73*149*2221*1459000305513721 |
| 112 | 1140593010 2594397055 2219 | $3*7^2*13*29*47*281*14503*10745088481$ |
| 113 | 1845518257 9303309636 6333 | 677*272602401466814027129 |
| 114 | 2986111268 1897706691 8552 | $2^3*37*113*229*797*9349*54833*95419$ |
| 115 | 4831629526 1201016328 4885 | 5*1381*28657*2441738887963981 |
| 116 | 7817740794 3098723020 3437 | 3*59*347*19489*514229*1270083883 |
| 117 | 1264937032 0429973934 88322 | 2*17*233*29717*135721*39589685693 |
| 118 | 2046711111 4739846236 91759 | 353*709*8969*336419*2710260697 |
| 119 | 3311648143 5169820171 80081 | 13*1597*159512939815855788121 |

# THE FIBONACCI NUMBERS

| n | FIBONACCI NUMBER AND FACTORIZATION |
|---|---|
| 120 | 5358359254 9909666408 71840 |
| | $2^5*3^2*5*7*11*23*31*41*61*241*2161*2521*20641$ |
| 121 | 8670007398 5079486580 51921 |
| | 89*97415813466381445596089 |
| 122 | 1402836665 3498915298 923761 |
| | 4513*555003497*5600748293801 |
| 123 | 2269837405 2006863956 975682 |
| | 2*2789*59369*68541957733949701 |
| 124 | 3672674070 5505779255 899443 |
| | 3*557*2417*3010349*3020733700601 |
| 125 | 5942511475 7512643212 875125 |
| | $5^3$*3001*158414167964045700001 |
| 126 | 9615185546 3018422468 774568 |
| | $2^3$*13*17*19*29*211*421*1009*31249*35239681 |
| 127 | 1555769702 2053106568 1649693 |
| | 27941*5568053048227732210073 |
| 128 | 2517288256 8354948815 0424261 |
| | 3*7*47*127*1087*2207*4481*186812208641 |
| 129 | 4073057959 0408055383 2073954 |
| | 2*257*5417*8513*39639893*433494437 |
| 130 | 6590346215 8763004198 2498215 |
| | 5*11*131*233*521*2081*24571*14736206161 |
| 131 | 1066340417 4917105958 14572169 |
| | 1066340417491710595814572169 |

THE FIBONACCI NUMBERS

| n | FIBONACCI NUMBER AND FACTORIZATION |
|---|---|
| 132 | 1725375039 0793406377 97070384 |
| | $2^4$*$3^2$*43*89*199*307*9901*19801*261399601 |
| 133 | 2791715456 5710512336 11642553 |
| | 13*37*113*3457*42293*351301301942501 |
| 134 | 4517090495 6503918714 08712937 |
| | 269*4021*116849*1429913*24994118449 |
| 135 | 7308805952 2214431050 20355490 |
| | 2*5*17*53*61*109*109441*1114769954367361 |
| 136 | 1182589644 7871834976 429068427 |
| | 3*7*67*1597*3571*63443*23230657239121 |
| 137 | 1913470240 0093278081 449423917 |
| | 19134702400093278081449423917 |
| 138 | 3096059884 7965113057 878492344 |
| | $2^3$*137*139*461*691*829*18077*28657*1485571 |
| 139 | 5009530124 8058391139 327916261 |
| | 277*2114537501*85526722937689093 |
| 140 | 8105590009 6023504197 206408605 |
| | 3*5*11*13*29*41*71*281*911*141961*12317523121 |
| 141 | 1311512013 4408189533 6534324866 |
| | 2*108289*1435097*142017737*2971215073 |
| 142 | 2122071014 4010539953 3740733471 |
| | 6673*46165371073*688846502588399 |
| 143 | 3433583027 8418729487 0275058337 |
| | 89*233*8581*1929584153756850496621 |

THE FIBONACCI NUMBERS

| n | FIBONACCI NUMBER AND FACTORIZATION |
|---|---|
| 144 | 5555654042 2429269440 4015791808 |
| | $2^6*3^3$*7*17*19*23*47*107*1103*103681*10749957121 |
| 145 | 8989237070 0847998927 4290850145 |
| | 5*514229*3496199969307370798902O1 |
| 146 | 1454489111 2327726836 7830664195 3 |
| | 151549*9375829*86020717*11899937029 |
| 147 | 2353412818 2412526729 5259749209 8 |
| | 2*13*97*293*421*3529*6168709*347502052673 |
| 148 | 3807901929 4740253566 3090413405 1 |
| | 3*73*149*2221*11987*54018521*81143477963 |
| 149 | 6161314747 7152780295 8350162614 9 |
| | 110557*162709*4000949*85607646594577 |
| 150 | 9969216677 1893033862 1440576020 0 |
| | $2^3*5^2$*11*31*61*101*151*3001*12301*18451*230686501 |

# THE LUCAS NUMBERS

In this table, primitive factors, that is, primes that divide a member of the sequence for the first time are underlined.

| n | LUCAS NUMBER | FACTORIZATION |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 3 | $\underline{3}$ |
| 3 | 4 | $\underline{2}^2$ |
| 4 | 7 | $\underline{7}$ |
| 5 | 11 | $\underline{11}$ |
| 6 | 18 | $2*3^2$ |
| 7 | 29 | $\underline{29}$ |
| 8 | 47 | $\underline{47}$ |
| 9 | 76 | $2^2*\underline{19}$ |
| 10 | 123 | $3*\underline{41}$ |
| 11 | 199 | $\underline{199}$ |
| 12 | 322 | $2*7*\underline{23}$ |
| 13 | 521 | $\underline{521}$ |
| 14 | 843 | $3*\underline{281}$ |
| 15 | 1364 | $2^2*11*\underline{31}$ |
| 16 | 2207 | $\underline{2207}$ |
| 17 | 3571 | $\underline{3571}$ |
| 18 | 5778 | $2*3^3*\underline{107}$ |
| 19 | 9349 | $\underline{9349}$ |
| 20 | 15127 | $7*\underline{2161}$ |
| 21 | 24476 | $2^2*29*\underline{211}$ |
| 22 | 39603 | $3*\underline{43}*\underline{307}$ |
| 23 | 64079 | $\underline{139}*\underline{461}$ |

THE LUCAS NUMBERS

| n | LUCAS NUMBER | FACTORIZATION |
|---|---|---|
| 24 | 103682 | 2*47*1103 |
| 25 | 167761 | 11*101*151 |
| 26 | 271443 | 3*90481 |
| 27 | 439204 | $2^2$*19*5779 |
| 28 | 710647 | $7^2$*14503 |
| 29 | 1149851 | 59*19489 |
| 30 | 1860498 | 2*$3^2$*41*2521 |
| 31 | 3010349 | 3010349 |
| 32 | 4870847 | 1087*4481 |
| 33 | 7881196 | $2^2$*199*9901 |
| 34 | 12752043 | 3*67*63443 |
| 35 | 20633239 | 11*29*71*911 |
| 36 | 33385282 | 2*7*23*103681 |
| 37 | 54018521 | 54018521 |
| 38 | 87403803 | 3*29134601 |
| 39 | 141422324 | $2^2$*79*521*859 |
| 40 | 228826127 | 47*1601*3041 |
| 41 | 370248451 | 370248451 |
| 42 | 599074578 | 2*$3^2$*83*281*1427 |
| 43 | 969323029 | 6709*144481 |
| 44 | 1568397607 | 7*263*881*967 |
| 45 | 2537720636 | $2^2$*11*19*31*181*541 |
| 46 | 4106118243 | 3*4969*275449 |
| 47 | 6643838879 | 6643838879 |

## THE LUCAS NUMBERS

| n | LUCAS NUMBER | FACTORIZATION |
|---|---|---|
| 48 | 1074995712 2 | 2*769*2207*3167 |
| 49 | 1739379600 1 | 29*599786069 |
| 50 | 2814375312 3 | 3*41*401*570601 |
| 51 | 4553754912 4 | $2^2$*919*3469*3571 |
| 52 | 7368130224 7 | 7*103*102193207 |
| 53 | 1192188513 71 | 119218851371 |
| 54 | 1929001536 18 | 2*$3^4$*107*11128427 |
| 55 | 3121190049 89 | $11^2$*199*331*39161 |
| 56 | 5050191586 07 | 47*10745088481 |
| 57 | 8171381635 96 | $2^2$*229*9349*95419 |
| 58 | 1322157322 203 | 3*347*1270083883 |
| 59 | 2139295485 799 | 709*8969*336419 |
| 60 | 3461452808 002 | 2*7*23*241*2161*20641 |
| 61 | 5600748293 801 | 5600748293801 |
| 62 | 9062201101 803 | 3*3020733700601 |
| 63 | 1466294939 5604 | $2^2$*19*29*211*1009*31249 |
| 64 | 2372515049 7407 | 127*186812208641 |
| 65 | 3838809989 3011 | 11*131*521*2081*24571 |
| 66 | 6211325039 0418 | 2*$3^2$*43*307*261399601 |
| 67 | 1005013502 83429 | 4021*24994118449 |
| 68 | 1626146006 73847 | 7*23230657239121 |
| 69 | 2631159509 57276 | $2^2$*139*461*691*1485571 |
| 70 | 4257305516 31123 | 3*41*281*12317523121 |
| 71 | 6888465025 88399 | 688846502588399 |

## THE LUCAS NUMBERS

| | | |
|---|---|---|
| 72 | 1114577054 219522 | 2*47*1103*10749957121 |
| 73 | 1803423556 807921 | 151549*11899937029 |
| 74 | 2918000611 027443 | 3*11987*81143477963 |
| 75 | 4721424167 835364 | $2^2$*11*31*101*151*12301*18451 |
| 76 | 7639424778 862807 | 7*1091346396980401 |
| 77 | 1236084894 6698171 | 29*199*229769*9321929 |
| 78 | 2000027372 5560978 | 2*$3^2$*90481*12280217041 |
| 79 | 3236112267 2259149 | 32361122672259149 |
| 80 | 5236139639 7820127 | 2207*23725145626561 |
| 81 | 8472251907 0079276 | $2^2$*19*3079*5779*62650261 |
| 82 | 1370839154 67899403 | 3*163*800483*350207569 |
| 83 | 2218064345 37978679 | 35761381*6202401259 |
| 84 | 3588903500 05878082 | 2*$7^2$*23*167*14503*65740583 |
| 85 | 5806967845 43856761 | 11*3571*1158551*12760031 |
| 86 | 9395871345 49734843 | 3*3131957115165782 81 |
| 87 | 1520283919 093591604 | $2^2$*59*349*19489*947104099 |
| 88 | 2459871053 643326447 | 47*93058241*562418561 |
| 89 | 3980154972 736918051 | 179*22235502640988369 |
| 90 | 6440026026 380244498 | 2*$3^3$*41*107*2521*10783342081 |
| 91 | 1042018099 9117162549 | 29*521*689667151970161 |
| 92 | 1686020702 5497407047 | 7*253367*9506372193863 |
| 93 | 2728038802 4614569596 | $2^2$*63799*3010349*35510749 |
| 94 | 4414059505 0111976643 | 3*563*5641*4632894751907 |
| 95 | 7142098307 4726546239 | 11*191*9349*41611*87382901 |

THE LUCAS NUMBERS

| n | LUCAS NUMBER | FACTORIZATION |
|---|---|---|
| 96 | 1155615781 2483852288 2 | 2*1087*4481*11862575248703 |
| 97 | 1869825611 9956506912 1 | 3299*56678557502141579 |
| 98 | 3025441393 2440359200 3 | 3*281*5881*61025309469041 |
| 99 | 4895267005 2396866112 4 | $2^2$*19*199*991*2179*9901*1513909 |
| 100 | 7920708398 4837225312 7 | 7*2161*9125201*5738108801 |
| 101 | 1281597540 3723409142 51 | 809*7879*201062946718741 |
| 102 | 2073668380 2207131673 78 | 2*$3^2$*67*409*63443*66265118449 |
| 103 | 3355265920 5930540816 29 | 619*1031*5257480026438961 |
| 104 | 5428934300 8137672490 07 | 47*3329*106513889*325759201 |
| 105 | 8784200221 4068213306 36 | $2^2$*11*29*31*71*211*911*21211*767131 |
| 106 | 1421313452 2220588579 643 | 3*1483*2969*1076012367720403 |
| 107 | 2299733474 3627409910 279 | 47927441*479836483312919 |
| 108 | 3721046926 5847998489 922 | 2*7*23*6263*103681*177962167367 |
| 109 | 6020780400 9475408400 201 | 128621*788071*593985111211 |
| 110 | 9741827327 5323406890 123 | 3*41*43*307*59996854928656801 |
| 111 | 1576260772 8479881529 0324 | $2^2$*4441*146521*1121101*54018521 |
| 112 | 2550443505 6012222218 0447 | 223*449*2207*1154149773784223 |
| 113 | 4126704278 4492103747 0771 | 412670427844921037470771 |
| 114 | 6677147784 0504325965 1218 | 2*$3^2$*227*26449*29134601*212067587 |
| 115 | 1080385206 2499642971 21989 | 11*139*461*1151*5981*324301*686551 |
| 116 | 1748099984 6550075567 73207 | 7*299281*834428410879506721 |
| 117 | 2828485190 9049718538 95196 | $2^2$*19*79*521*859*1052645985555841 |
| 118 | 4576585175 5599794106 68403 | 3*15247723*100049587197598387 |
| 119 | 7405070366 4649512645 63599 | 29*239*3571*10711*27932732439809 |

THE LUCAS NUMBERS

| n | LUCAS NUMBER AND FACTORIZATION |
|---|---|
| 120 | 1198165554 2024930675 232002 |
| | 2*47*1103*1601*3041*23735900452321 |
| 121 | 1938672590 8489881939 795601 |
| | 199*97420733208491869044199 |
| 122 | 3136838145 0514812615 027603 |
| | 3*19763*21291929*24848660119363 |
| 123 | 5075510735 9004694554 823204 |
| | $2^2$*4767481*370248451*7188487771 |
| 124 | 8212348880 9519507169 850807 |
| | 7*743*467729*33758740830460183 |
| 125 | 1328785961 6852420172 4674011 |
| | 11*101*151*251*112128001*28143378001 |
| 126 | 2150020849 7804370889 4524818 |
| | 2*$3^3$*83*107*281*1427*1461601*764940961 |
| 127 | 3478806811 4656791061 9198829 |
| | 509*5081*487681*13822681*19954241 |
| 128 | 5628827661 2461161951 3723647 |
| | 119809*4698167634523379875583 |
| 129 | 9107634472 7117953013 2922476 |
| | $2^2$*6709*144481*308311*761882591401 |
| 130 | 1473646213 3957911496 46646123 |
| | 3*41*3121*90481*42426476041450801 |
| 131 | 2384409660 6669706797 79568599 |
| | 1049*414988698461*5477332620091 |

## THE LUCAS NUMBERS

| n | LUCAS NUMBER AND FACTORIZATION |
| --- | --- |
| 132 | 3858055874 0627618294 26214722 |
| | 2*7*23*263*881*967*5281*66529*152204449 |
| 133 | 6242465534 7297325092 05783321 |
| | 29*9349*10694421739*2152958650459 |
| 134 | 1010052140 8792494338 631998043 |
| | 3*6163*201912469249*2705622682163 |
| 135 | 1634298694 3522226847 837781364 |
| | $2^2$*11*19*31*181*271*541*811*5779*42391*119611 |
| 136 | 2644350835 2314721186 469779407 |
| | 47*56262783728329194013765488l |
| 137 | 4278649529 5836948034 307560771 |
| | 541721291*78982487870939058281 |
| 138 | 6923000364 8151669220 777340178 |
| | 2*$3^2$*4969*16561*162563*275449*1043766587 |
| 139 | 1120164989 4398861725 5084900949 |
| | 30859*253279129*14331800109223159 |
| 140 | 1812465025 9214028647 5862241127 |
| | $7^2$*2161*14503*118021448662479038881 |
| 141 | 2932630015 3612890373 0947142076 |
| | $2^2$*79099591*6643838879*139509555271 |
| 142 | 4745095041 2826919020 6809383203 |
| | 3*283*569*2820403*9799987*35537616083 |
| 143 | 7677725056 6439809393 7756525279 |
| | 199*521*1957099*2120119*1784714380021 |

THE LUCAS NUMBERS

| n | LUCAS NUMBER AND FACTORIZATION |
| --- | --- |
| 144 | 1242282009 7926672841 4456590848 2 |
| | 2*769*2207*3167*115561578124838522881 |
| 145 | 2010054515 4570653780 8232243376 1 |
| | 11*59*19489*120196353941*1322154751061 |
| 146 | 3252336525 2497326622 2688834224 3 |
| | 3*29201*37125857850184727260788881 |
| 147 | 5262391040 7067980403 0921077600 4 |
| | $2^2$*29*211*65269*620929*8844991*599786069 |
| 148 | 8514727565 9565307025 3609911824 7 |
| | 7*10661921*114087288048701953998401 |
| 149 | 1377711860 6663328742 8453098942 51 |
| | 952111*4434539*3263039535803245519 |
| 150 | 2229184617 2619859445 3814090124 98 |
| | 2*$3^2$*41*401*601*2521*570601*87129547172401 |

SQUARES AND SUMS OF SQUARES OF FIBONACCI NUMBERS

The sum of the squares of the Fibonacci numbers from 1 to n is given by $F_n F_{n+1}$.

| n | $F_n^2$ | $\Sigma F_n^2$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 2 |
| 3 | 4 | 6 |
| 4 | 9 | 15 |
| 5 | 25 | 40 |
| 6 | 64 | 104 |
| 7 | 169 | 273 |
| 8 | 441 | 714 |
| 9 | 1156 | 1870 |
| 10 | 3025 | 4895 |
| 11 | 7921 | 12816 |
| 12 | 20736 | 33552 |
| 13 | 54289 | 87841 |
| 14 | 142129 | 229970 |
| 15 | 372100 | 602070 |
| 16 | 974169 | 1576239 |
| 17 | 2550409 | 4126648 |
| 18 | 6677056 | 10803704 |
| 19 | 17480761 | 28284465 |
| 20 | 45765225 | 74049690 |
| 21 | 119814916 | 193864606 |
| 22 | 313679521 | 507544127 |
| 23 | 821223649 | 1328767776 |
| 24 | 2149991424 | 3478759200 |
| 25 | 5628750625 | 9107509825 |
| 26 | 1473626044 9 | 2384377027 4 |
| 27 | 3858003072 4 | 6242380099 8 |
| 28 | 1010038317 21 | 1634276327 19 |
| 29 | 2644314644 41 | 4278590971 60 |
| 30 | 6922905616 00 | 1120149658 760 |
| 31 | 1812440220 361 | 2932589879 121 |
| 32 | 4745030099 481 | 7677619978 602 |
| 33 | 1242265007 8084 | 2010027005 6686 |
| 34 | 3252292013 4769 | 5262319019 1455 |
| 35 | 8514611032 6225 | 1377693005 17680 |
| 36 | 2229154108 43904 | 3606847113 61584 |
| 37 | 5836001222 05489 | 9442848335 67073 |
| 38 | 1527884955 772561 | 2472169789 339634 |
| 39 | 4000054745 112196 | 6472224534 451830 |
| 40 | 1047227927 9564025 | 1694450381 4015855 |

## CUBES AND SUMS OF CUBES OF FIBONACCI NUMBERS

The sum of the cubes of the Fibonacci numbers from 1 to n is given by

$$\frac{F_{n+1}^2 F_n + (-1)^{n+1} F_{n-1} + 1}{2}$$

| n | $F_n^3$ | $\Sigma F_n^3$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 2 |
| 3 | 8 | 10 |
| 4 | 27 | 37 |
| 5 | 125 | 162 |
| 6 | 512 | 674 |
| 7 | 2197 | 2871 |
| 8 | 9261 | 12132 |
| 9 | 39304 | 51436 |
| 10 | 166375 | 217811 |
| 11 | 704969 | 922780 |
| 12 | 2985984 | 3908764 |
| 13 | 12649337 | 16558101 |
| 14 | 53582633 | 70140734 |
| 15 | 226981000 | 297121734 |
| 16 | 961504803 | 1258626537 |
| 17 | 4073003173 | 5331629710 |
| 18 | 1725351270 4 | 2258514241 4 |
| 19 | 7308706174 1 | 9567220415 5 |
| 20 | 3096017471 25 | 4052739512 80 |
| 21 | 1311494070 536 | 1716768021 816 |
| 22 | 5555577996 431 | 7272346018 247 |
| 23 | 2353380610 9393 | 3080615212 7640 |
| 24 | 9969080234 8032 | 1304969544 75672 |
| 25 | 4222970156 40625 | 5527939701 16297 |
| 26 | 1788878864 685457 | 2341672834 801754 |
| 27 | 7577812474 746632 | 9919485309 548386 |
| 28 | 3210012876 3082731 | 4201961407 2631117 |
| 29 | 1359783275 28030989 | 1779979416 00662106 |
| 30 | 5760134388 73664000 | 7540113804 74326106 |
| 31 | 2440032083 025183109 | 3194043463 499509215 |
| 32 | 1033614177 0970357629 | 1353018523 4469866844 |
| 33 | 4378459916 6913148552 | 5731478440 1383015396 |
| 34 | 1854745384 3861237810 3 | 2427893228 3999539349 9 |
| 35 | 7856827529 2137976962 5 | 1028472075 7613751631 24 |

## FOURTH POWERS AND SUMS OF FOURTH POWERS OF FIBONACCI NUMBERS

The sum of the fourth powers of the first n terms of the Fibonacci sequence is given by

$$\frac{F_{2n+1}L_{n-1}L_{n+2} + 6n + 3}{25}$$

| n | $F_n^4$ | $\Sigma F_n^4$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 2 |
| 3 | 16 | 18 |
| 4 | 81 | 99 |
| 5 | 625 | 724 |
| 6 | 4096 | 4820 |
| 7 | 28561 | 33381 |
| 8 | 194481 | 227862 |
| 9 | 1336336 | 1564198 |
| 10 | 9150625 | 10714823 |
| 11 | 62742241 | 73457064 |
| 12 | 429981696 | 503438760 |
| 13 | 2947295521 | 3450734281 |
| 14 | 2020065264 1 | 2365138692 2 |
| 15 | 1384584100 00 | 1621097969 22 |
| 16 | 9490052405 61 | 1111115037 483 |
| 17 | 6504586067 281 | 7615701104 764 |
| 18 | 4458307682 7136 | 5219877793 1900 |
| 19 | 3055770051 39121 | 3577757830 71021 |
| 20 | 2094455819 300625 | 2452231602 371646 |
| 21 | 1435561409 6087056 | 1680784569 8458702 |
| 22 | 9839484189 4789441 | 1152026875 93248143 |
| 23 | 6744082816 76875201 | 7896109692 70123344 |
| 24 | 4622463123 273547776 | 5412074092 543671120 |
| 25 | 3168283359 8437890625 | 3709490769 0981561745 |

SQUARES AND SUMS OF SQUARES OF LUCAS NUMBERS

The sum of the squares of the first n Lucas numbers is given by

$$L_n L_{n+1} - 2$$

| n | $L_n^2$ | $\Sigma L_n^2$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 9 | 10 |
| 3 | 16 | 26 |
| 4 | 49 | 75 |
| 5 | 121 | 196 |
| 6 | 324 | 520 |
| 7 | 841 | 1361 |
| 8 | 2209 | 3570 |
| 9 | 5776 | 9346 |
| 10 | 15129 | 24475 |
| 11 | 39601 | 64076 |
| 12 | 103684 | 167760 |
| 13 | 271441 | 439201 |
| 14 | 710649 | 1149850 |
| 15 | 1860496 | 3010346 |
| 16 | 4870849 | 7881195 |
| 17 | 12752041 | 20633236 |
| 18 | 33385284 | 54018520 |
| 19 | 87403801 | 141422321 |
| 20 | 228826129 | 370248450 |
| 21 | 599074576 | 969323026 |
| 22 | 1568397609 | 2537720635 |
| 23 | 4106118241 | 6643838876 |
| 24 | 1074995712 4 | 1739379600 0 |
| 25 | 2814375312 1 | 4553754912 1 |
| 26 | 7368130224 9 | 1192188513 70 |
| 27 | 1929001536 16 | 3121190049 86 |
| 28 | 5050191586 09 | 8171381635 95 |
| 29 | 1322157322 201 | 2139295485 796 |
| 30 | 3461452808 004 | 5600748293 800 |
| 31 | 9062201101 801 | 1466294939 5601 |
| 32 | 2372515049 7409 | 3838809989 3010 |
| 33 | 6211325039 0416 | 1005013502 83426 |
| 34 | 1626146006 73849 | 2631159509 57275 |
| 35 | 4257305516 31121 | 6888465025 88396 |
| 36 | 1114577054 219524 | 1803423556 807920 |
| 37 | 2918000611 027441 | 4721424167 835361 |
| 38 | 7639424778 862809 | 1236084894 6698170 |
| 39 | 2000027372 5560976 | 3236112267 2259146 |
| 40 | 5236139639 7820129 | 8472251907 0079275 |

CUBES AND SUMS OF CUBES OF LUCAS NUMBERS

The sum of the cubes of the first n Lucas numbers is given by

$$\frac{L_{3n+2} - 3}{2} + 3(-1)^n L_{n-1} + 3$$

| n | $L_n^3$ | $\Sigma L_n^3$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 27 | 28 |
| 3 | 64 | 92 |
| 3 | 343 | 435 |
| 4 | 1331 | 1766 |
| 6 | 5832 | 7598 |
| 7 | 24389 | 31987 |
| 8 | 103823 | 135810 |
| 9 | 438976 | 574786 |
| 10 | 1860867 | 2435653 |
| 11 | 7880599 | 10316252 |
| 12 | 33386248 | 43702500 |
| 13 | 141420761 | 185123261 |
| 14 | 599077107 | 784200368 |
| 15 | 2537716544 | 3321916912 |
| 16 | 1074996374 3 | 1407188065 5 |
| 17 | 4553753841 1 | 5960941906 6 |
| 18 | 1929001709 52 | 2525095900 18 |
| 19 | 8171381355 49 | 1069647725 567 |
| 20 | 3461452853 383 | 4531100578 950 |
| 21 | 1466294932 2176 | 1919404990 1126 |
| 22 | 6211325050 9227 | 8130730041 0353 |
| 23 | 2631159507 65039 | 3444232511 75392 |
| 24 | 1114577054 530568 | 1459000305 705960 |
| 25 | 4721424167 332081 | 6180424473 038041 |
| 26 | 2000027372 6375307 | 2618069819 9413348 |
| 27 | 8472251906 8761664 | 1109032172 68175012 |
| 28 | 3588903500 08010023 | 4697935672 76185035 |
| 29 | 1520283919 090142051 | 1990077486 366327086 |
| 30 | 6440026026 385825992 | 8430103512 752153078 |
| 31 | 2728038802 4605538549 | 3571049153 7357691627 |
| 32 | 1155615781 2485313542 3 | 1512720696 6221082705 0 |
| 33 | 4895267005 2394501753 6 | 6407987701 8615584458 6 |
| 34 | 2073668380 2207514235 07 | 2714467150 4069072680 93 |
| 35 | 8784200221 4067594309 19 | 1149866737 1813666699 012 |

# FOURTH POWERS AND SUMS OF FOURTH POWERS OF LUCAS NUMBERS

The sum of the fourth powers of the first n Lucas numbers is given by

$$F_{4n+2} + 4(-1)^n F_{2n+1} + 6n - 5$$

| n | $L_n^4$ | $\Sigma L_n^4$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 81 | 82 |
| 3 | 256 | 338 |
| 4 | 2401 | 2739 |
| 5 | 14641 | 17380 |
| 6 | 104976 | 122356 |
| 7 | 707281 | 829637 |
| 8 | 4879681 | 5709318 |
| 9 | 33362176 | 39071494 |
| 10 | 228886641 | 267958135 |
| 11 | 1568239201 | 1836197336 |
| 12 | 1075037185 6 | 1258656919 2 |
| 13 | 7368021648 1 | 8626678567 3 |
| 14 | 5050220012 01 | 5912887868 74 |
| 15 | 3461445366 016 | 4052734152 890 |
| 16 | 2372516998 0801 | 2777790413 3691 |
| 17 | 1626145496 65681 | 1903924537 99372 |
| 18 | 1114577187 760656 | 1304969641 560028 |
| 19 | 7639424429 247601 | 8944394070 807629 |
| 20 | 5236139731 3124641 | 6130579138 3932270 |
| 21 | 3588903476 09579776 | 4201961389 93512046 |
| 22 | 2459871059 916916881 | 2880067198 910428927 |
| 23 | 1686020700 9072934081 | 1974027420 7983363008 |
| 24 | 1155615781 6783835137 6 | 1353018523 7582171438 4 |
| 25 | 7920708397 3579724064 1 | 9273726921 1161895502 5 |

# FIBONACCI PRIMES

| n | $F_n$ |
|---|---|
| 3 | 2 |
| 4 | 3 |
| 5 | 5 |
| 7 | 13 |
| 11 | 89 |
| 13 | 233 |
| 17 | 1597 |
| 23 | 28657 |
| 29 | 514229 |
| 43 | 433494437 |
| 47 | 2971215073 |
| 83 | 9919485309 4755497 |
| 131 | 1066340417 4917105958 14752169 |
| 137 | 1913470240 0093278081 449423917 |
| 359 | 4754204377 3469822074 7368027166 7493829277 0141701655 7193662268<br>7163769354 76241 |
| 431 | 5298927110 0609562179 2039556787 7846701971 1275902953 4506620905<br>1628347699 5513442468 9676262369 |
| 433 | 1387277127 8047838271 1418610318 6246392258 4503581717 8369007991<br>8032136025 2259546025 9371256835 3 |
| 449 | 3061719992 4845450305 5431384808 3717208111 2854323537 3849713167<br>4799321571 2381490159 3344280566 5949 |
| 509 | 1059799926 5301490732 5996436715 0500341251 5860435409 4219325600<br>0968014297 4347195483 1402932543 9619576987 6129909 |
| 569 | 3668447431 6080978061 4736136462 7563045110 0586901195 2298152702<br>4286841776 8061193560 8579043350 1787954051 5228143777 781065869 |
| 571 | 9604120061 8922553823 9428833609 2486502610 4917411877 0678168222<br>6478902901 4378308478 8641925890 8418525433 1637646183 008074629 |

# LUCAS PRIMES

| $n$ | $L_n$ |
|---|---|
| 2 | 3 |
| 4 | 7 |
| 5 | 11 |
| 7 | 29 |
| 8 | 47 |
| 11 | 199 |
| 13 | 521 |
| 16 | 2207 |
| 17 | 3571 |
| 19 | 9349 |
| 31 | 3010349 |
| 37 | 54018521 |
| 41 | 370248451 |
| 47 | 6643838879 |
| 53 | 1192188513 71 |
| 61 | 5600748293 801 |
| 71 | 6888465025 88399 |
| 79 | 3236112267 2259149 |
| 113 | 4126704278 4492103747 0771 |
| 313 | 2588996112 0330341872 1656157249 4455300468 3007304420 1152332257 717521 |
| 353 | 5924299531 3457729780 5108237673 5473079828 6848921481 3748742645 3470557362 8371 |

FIBONACCI ENTRY POINTS

In this table Z(p) is the entry point of the given prime p in the Fibonacci sequence, that is, Z(p) is the subscript of the first Fibonacci number of which p is a divisor.

From Z(p) it is possible to obtain the period k(p) of the Fibonacci sequence modulo p according to the following rules.

(1) If Z(p) is odd, k(p) = 4 Z(p).

(2) If Z(p) = 2(2m+1), k(p) = Z(p).

(3) If $Z(p) = 2^k (2m+1)$, $k \geq 2$, k(p) = 2 Z(p).

The entry point of primes in the Lucas sequence is half the entry point in the Fibonacci sequence if Z(p) is even. If Z(p) is odd, the prime does not enter the Lucas sequence.

The period of the Lucas sequence modulo p is the same as the period of the Fibonacci sequence except for 5 for which the period is 4 instead of 20 in the Lucas sequence.

| p | Z(p) | p | Z(p) | p | Z(p) | p | Z(p) |
|---|---|---|---|---|---|---|---|
| 2 | 3 | 101 | 50 | 233 | 13 | 383 | 384 |
| 3 | 4 | 103 | 104 | 239 | 238 | 389 | 97 |
| 5 | 5 | 107 | 36 | 241 | 120 | 397 | 199 |
| 7 | 8 | 109 | 27 | 251 | 250 | 401 | 100 |
| 11 | 10 | 113 | 19 | 257 | 129 | 409 | 204 |
| 13 | 7 | 127 | 128 | 263 | 88 | 419 | 418 |
| 17 | 9 | 131 | 130 | 269 | 67 | 421 | 21 |
| 19 | 18 | 137 | 69 | 271 | 270 | 431 | 430 |
| 23 | 24 | 139 | 46 | 277 | 139 | 433 | 217 |
| 29 | 14 | 149 | 37 | 281 | 28 | 439 | 438 |
| 31 | 30 | 151 | 50 | 283 | 284 | 443 | 444 |
| 37 | 19 | 157 | 79 | 293 | 147 | 449 | 224 |
| 41 | 20 | 163 | 164 | 307 | 44 | 457 | 229 |
| 43 | 44 | 167 | 168 | 311 | 310 | 461 | 46 |
| 47 | 16 | 173 | 87 | 313 | 157 | 463 | 464 |
| 53 | 27 | 179 | 178 | 317 | 159 | 467 | 468 |
| 59 | 58 | 181 | 90 | 331 | 110 | 479 | 478 |
| 61 | 15 | 191 | 190 | 337 | 169 | 487 | 488 |
| 67 | 68 | 193 | 97 | 347 | 116 | 491 | 490 |
| 71 | 70 | 197 | 99 | 349 | 174 | 499 | 498 |
| 73 | 37 | 199 | 22 | 353 | 59 | 503 | 504 |
| 79 | 78 | 211 | 42 | 359 | 358 | 509 | 254 |
| 83 | 84 | 223 | 224 | 367 | 368 | 521 | 26 |
| 89 | 11 | 227 | 228 | 373 | 187 | 523 | 524 |
| 97 | 49 | 229 | 114 | 379 | 378 | 541 | 90 |

# FIBONACCI ENTRY POINTS

| p | Z(p) | p | Z(p) | p | Z(p) | p | Z(p) |
|---|---|---|---|---|---|---|---|
| 547 | 548 | 839 | 838 | 1153 | 577 | 1487 | 1488 |
| 557 | 31 | 853 | 427 | 1163 | 1164 | 1489 | 744 |
| 563 | 188 | 857 | 429 | 1171 | 1170 | 1493 | 747 |
| 569 | 284 | 859 | 78 | 1181 | 295 | 1499 | 1498 |
| 571 | 570 | 863 | 864 | 1187 | 1188 | 1511 | 302 |
| 577 | 289 | 877 | 439 | 1193 | 597 | 1523 | 508 |
| 587 | 588 | 881 | 88 | 1201 | 600 | 1531 | 1530 |
| 593 | 297 | 883 | 884 | 1213 | 607 | 1543 | 1544 |
| 599 | 598 | 887 | 888 | 1217 | 203 | 1549 | 774 |
| 601 | 300 | 907 | 908 | 1223 | 408 | 1553 | 259 |
| 607 | 608 | 911 | 70 | 1229 | 614 | 1559 | 1558 |
| 613 | 307 | 919 | 102 | 1231 | 410 | 1567 | 1568 |
| 617 | 309 | 929 | 464 | 1237 | 619 | 1571 | 1570 |
| 619 | 206 | 937 | 469 | 1249 | 312 | 1579 | 526 |
| 631 | 630 | 941 | 470 | 1259 | 1258 | 1583 | 1584 |
| 641 | 320 | 947 | 948 | 1277 | 213 | 1597 | 17 |
| 643 | 644 | 953 | 53 | 1279 | 426 | 1601 | 80 |
| 647 | 648 | 967 | 88 | 1283 | 1284 | 1607 | 1608 |
| 653 | 327 | 971 | 970 | 1289 | 322 | 1609 | 804 |
| 659 | 658 | 977 | 163 | 1291 | 430 | 1613 | 807 |
| 661 | 55 | 983 | 984 | 1297 | 649 | 1619 | 1618 |
| 673 | 337 | 991 | 198 | 1301 | 325 | 1621 | 810 |
| 677 | 113 | 997 | 499 | 1303 | 1304 | 1627 | 1628 |
| 683 | 684 | 1009 | 126 | 1307 | 436 | 1637 | 819 |
| 691 | 138 | 1013 | 507 | 1319 | 1318 | 1657 | 829 |
| 701 | 175 | 1019 | 1018 | 1321 | 660 | 1663 | 1664 |
| 709 | 118 | 1021 | 510 | 1327 | 1328 | 1667 | 1668 |
| 719 | 718 | 1031 | 206 | 1361 | 340 | 1669 | 834 |
| 727 | 728 | 1033 | 517 | 1367 | 1368 | 1693 | 847 |
| 733 | 367 | 1039 | 1038 | 1373 | 687 | 1697 | 849 |
| 739 | 738 | 1049 | 262 | 1381 | 115 | 1699 | 566 |
| 743 | 248 | 1051 | 1050 | 1399 | 1398 | 1709 | 854 |
| 751 | 750 | 1061 | 530 | 1409 | 352 | 1721 | 430 |
| 757 | 379 | 1063 | 1064 | 1423 | 1424 | 1723 | 1724 |
| 761 | 95 | 1069 | 89 | 1427 | 84 | 1733 | 289 |
| 769 | 96 | 1087 | 64 | 1429 | 357 | 1741 | 870 |
| 773 | 387 | 1091 | 1090 | 1433 | 717 | 1747 | 1748 |
| 787 | 788 | 1093 | 547 | 1439 | 1438 | 1753 | 877 |
| 797 | 57 | 1097 | 183 | 1447 | 1448 | 1759 | 1758 |
| 809 | 202 | 1103 | 48 | 1451 | 1450 | 1777 | 889 |
| 811 | 270 | 1109 | 554 | 1453 | 727 | 1783 | 1784 |
| 821 | 205 | 1117 | 559 | 1459 | 1458 | 1787 | 1788 |
| 823 | 824 | 1123 | 1124 | 1471 | 490 | 1789 | 894 |
| 827 | 828 | 1129 | 564 | 1481 | 740 | 1801 | 900 |
| 829 | 69 | 1151 | 230 | 1483 | 212 | 1811 | 1810 |

## FIBONACCI ENTRY POINTS

| p | Z(p) | p | Z(p) | p | Z(p) | p | Z(p) |
|---|---|---|---|---|---|---|---|
| 1823 | 608 | 2161 | 40 | 2539 | 2538 | 2861 | 1430 |
| 1831 | 1830 | 2179 | 198 | 2543 | 2544 | 2879 | 2878 |
| 1847 | 1848 | 2203 | 2204 | 2549 | 637 | 2887 | 2888 |
| 1861 | 930 | 2207 | 32 | 2551 | 2550 | 2897 | 1449 |
| 1867 | 1868 | 2213 | 1107 | 2557 | 1279 | 2903 | 2904 |
| 1871 | 374 | 2221 | 37 | 2579 | 2578 | 2909 | 727 |
| 1873 | 937 | 2237 | 373 | 2591 | 518 | 2917 | 1459 |
| 1877 | 313 | 2239 | 746 | 2593 | 1297 | 2927 | 2928 |
| 1879 | 1878 | 2243 | 748 | 2609 | 1304 | 2939 | 2938 |
| 1889 | 944 | 2251 | 750 | 2617 | 1309 | 2953 | 1477 |
| 1901 | 475 | 2267 | 756 | 2621 | 1310 | 2957 | 1479 |
| 1907 | 1908 | 2269 | 81 | 2633 | 1317 | 2963 | 2964 |
| 1913 | 319 | 2273 | 1137 | 2647 | 2648 | 2969 | 212 |
| 1931 | 1930 | 2281 | 380 | 2657 | 1329 | 2971 | 2970 |
| 1933 | 967 | 2287 | 2288 | 2659 | 886 | 2999 | 2998 |
| 1949 | 487 | 2293 | 1147 | 2663 | 888 | 3001 | 25 |
| 1951 | 390 | 2297 | 1149 | 2671 | 2670 | 3011 | 3010 |
| 1973 | 329 | 2309 | 577 | 2677 | 1339 | 3019 | 3018 |
| 1979 | 1978 | 2311 | 2310 | 2683 | 2684 | 3023 | 432 |
| 1987 | 1988 | 2333 | 389 | 2687 | 896 | 3037 | 1519 |
| 1993 | 997 | 2339 | 2338 | 2689 | 448 | 3041 | 80 |
| 1997 | 999 | 2341 | 585 | 2693 | 1347 | 3049 | 762 |
| 1999 | 666 | 2347 | 2348 | 2699 | 2698 | 3061 | 765 |
| 2003 | 2004 | 2351 | 2350 | 2707 | 2708 | 3067 | 3068 |
| 2011 | 2010 | 2357 | 1179 | 2711 | 2710 | 3079 | 162 |
| 2017 | 1009 | 2371 | 790 | 2713 | 1357 | 3083 | 3084 |
| 2027 | 676 | 2377 | 1189 | 2719 | 2718 | 3089 | 772 |
| 2029 | 1014 | 2381 | 595 | 2729 | 682 | 3109 | 777 |
| 2039 | 2038 | 2383 | 2384 | 2731 | 390 | 3119 | 3118 |
| 2053 | 1027 | 2389 | 398 | 2741 | 685 | 3121 | 260 |
| 2063 | 2064 | 2393 | 1197 | 2749 | 229 | 3137 | 1569 |
| 2069 | 1034 | 2399 | 2398 | 2753 | 459 | 3163 | 3164 |
| 2081 | 130 | 2411 | 2410 | 2767 | 2768 | 3167 | 96 |
| 2083 | 2084 | 2417 | 31 | 2777 | 463 | 3169 | 792 |
| 2087 | 2088 | 2423 | 2424 | 2789 | 41 | 3181 | 795 |
| 2089 | 261 | 2437 | 1219 | 2791 | 2790 | 3187 | 3188 |
| 2099 | 2098 | 2441 | 305 | 2797 | 1399 | 3191 | 3190 |
| 2111 | 2110 | 2447 | 816 | 2801 | 700 | 3203 | 3204 |
| 2113 | 1057 | 2459 | 2458 | 2803 | 2804 | 3209 | 1604 |
| 2129 | 1064 | 2467 | 2468 | 2819 | 2818 | 3217 | 1609 |
| 2131 | 2130 | 2473 | 1237 | 2833 | 1417 | 3221 | 805 |
| 2137 | 1069 | 2477 | 1239 | 2837 | 1419 | 3229 | 807 |
| 2141 | 535 | 2503 | 2504 | 2843 | 2844 | 3251 | 650 |
| 2143 | 2144 | 2521 | 60 | 2851 | 2850 | 3253 | 1627 |
| 2153 | 1077 | 2531 | 2530 | 2857 | 1429 | 3257 | 1629 |

## FIBONACCI ENTRY POINTS

| p | Z(p) | p | Z(p) | p | Z(p) | p | Z(p) |
|---|---|---|---|---|---|---|---|
| 3259 | 3258 | 3617 | 603 | 4001 | 1000 | 4363 | 4364 |
| 3271 | 1090 | 3623 | 1208 | 4003 | 4004 | 4373 | 81 |
| 3299 | 194 | 3631 | 1210 | 4007 | 4008 | 4391 | 878 |
| 3301 | 1650 | 3637 | 1819 | 4013 | 223 | 4397 | 2199 |
| 3307 | 3308 | 3643 | 3644 | 4019 | 4018 | 4409 | 2204 |
| 3313 | 1657 | 3659 | 3658 | 4021 | 134 | 4421 | 1105 |
| 3319 | 3318 | 3671 | 3670 | 4027 | 4028 | 4423 | 4424 |
| 3323 | 1108 | 3673 | 1837 | 4049 | 1012 | 4441 | 222 |
| 3329 | 208 | 3677 | 1839 | 4051 | 1350 | 4447 | 4448 |
| 3331 | 1110 | 3691 | 3690 | 4057 | 2029 | 4451 | 4450 |
| 3343 | 3344 | 3697 | 1849 | 4073 | 2037 | 4457 | 2229 |
| 3347 | 1116 | 3701 | 925 | 4079 | 4078 | 4463 | 4464 |
| 3359 | 3358 | 3709 | 927 | 4091 | 4090 | 4481 | 64 |
| 3361 | 1680 | 3719 | 3718 | 4093 | 2047 | 4483 | 4484 |
| 3371 | 3370 | 3727 | 3728 | 4099 | 4098 | 4493 | 749 |
| 3373 | 1687 | 3733 | 1867 | 4111 | 4110 | 4507 | 4508 |
| 3389 | 1694 | 3739 | 534 | 4127 | 4128 | 4513 | 61 |
| 3391 | 1130 | 3961 | 1880 | 4129 | 2064 | 4517 | 2259 |
| 3407 | 1136 | 3767 | 1256 | 4133 | 689 | 4519 | 4518 |
| 3413 | 1707 | 3769 | 1884 | 4139 | 4138 | 4523 | 4524 |
| 3433 | 1717 | 3779 | 3778 | 4153 | 2077 | 4547 | 1516 |
| 3449 | 1724 | 3793 | 1897 | 4157 | 297 | 4549 | 2274 |
| 3457 | 133 | 3797 | 1899 | 4159 | 1386 | 4561 | 380 |
| 3461 | 1730 | 3803 | 3804 | 4177 | 2089 | 4567 | 4568 |
| 3463 | 3464 | 3821 | 1910 | 4201 | 525 | 4583 | 1528 |
| 3467 | 1156 | 3823 | 3824 | 4211 | 4210 | 4591 | 4590 |
| 3469 | 102 | 3833 | 1917 | 4217 | 2109 | 4597 | 2299 |
| 3491 | 3490 | 3847 | 3848 | 4219 | 4218 | 4603 | 4604 |
| 3499 | 1166 | 3851 | 770 | 4229 | 2114 | 4621 | 462 |
| 3511 | 3510 | 3853 | 1927 | 4231 | 1410 | 4637 | 773 |
| 3517 | 1759 | 3863 | 3864 | 4241 | 2120 | 4639 | 4638 |
| 3527 | 3528 | 3877 | 1939 | 4243 | 4244 | 4643 | 1548 |
| 3529 | 147 | 3881 | 485 | 4253 | 2127 | 4649 | 1162 |
| 3533 | 589 | 3889 | 1944 | 4259 | 4258 | 4651 | 930 |
| 3539 | 3538 | 3907 | 3908 | 4261 | 355 | 4657 | 2329 |
| 3541 | 885 | 3911 | 3910 | 4271 | 4270 | 4663 | 4664 |
| 3547 | 3548 | 3917 | 1959 | 4273 | 2137 | 4673 | 2337 |
| 3557 | 593 | 3919 | 1306 | 4283 | 612 | 4679 | 4678 |
| 3559 | 3558 | 3923 | 3924 | 4289 | 536 | 4691 | 938 |
| 3571 | 34 | 3929 | 982 | 4297 | 2149 | 4703 | 1568 |
| 3581 | 1790 | 3931 | 3930 | 4327 | 4328 | 4721 | 2360 |
| 3583 | 3584 | 3943 | 3944 | 4337 | 2169 | 4723 | 4724 |
| 3593 | 1797 | 3947 | 1316 | 4339 | 4338 | 4729 | 591 |
| 3607 | 3608 | 3967 | 3968 | 4349 | 1087 | 4733 | 263 |
| 3613 | 1807 | 3989 | 1994 | 4357 | 2179 | 4751 | 950 |

# FIBONACCI ENTRY POINTS

| p | Z(p) | p | Z(p) | p | Z(p) | p | Z(p) |
|---|---|---|---|---|---|---|---|
| 4759 | 1586 | 5147 | 572 | 5527 | 5528 | 5897 | 983 |
| 4783 | 4784 | 5153 | 2577 | 5531 | 1106 | 5903 | 5904 |
| 4787 | 4788 | 5167 | 5168 | 5557 | 2779 | 5923 | 5924 |
| 4789 | 2394 | 5171 | 5170 | 5563 | 5564 | 5927 | 5928 |
| 4793 | 2397 | 5179 | 5178 | 5569 | 2784 | 5939 | 5938 |
| 4799 | 4798 | 5189 | 1297 | 5573 | 2787 | 5953 | 2977 |
| 4801 | 2400 | 5197 | 2599 | 5581 | 465 | 5981 | 230 |
| 4813 | 2407 | 5209 | 651 | 5591 | 1118 | 5987 | 5988 |
| 4817 | 2409 | 5227 | 5228 | 5623 | 5624 | 6007 | 6008 |
| 4831 | 690 | 5231 | 5230 | 5639 | 5638 | 6011 | 1202 |
| 4861 | 810 | 5233 | 2617 | 5641 | 188 | 6029 | 1507 |
| 4871 | 974 | 5237 | 873 | 5647 | 5648 | 6037 | 3019 |
| 4877 | 2439 | 5261 | 2630 | 5651 | 5650 | 6043 | 6044 |
| 4889 | 2444 | 5273 | 2637 | 5653 | 257 | 6047 | 6048 |
| 4903 | 4904 | 5279 | 5278 | 5657 | 2829 | 6053 | 3027 |
| 4909 | 409 | 5281 | 264 | 5659 | 5658 | 6067 | 6068 |
| 4919 | 4918 | 5297 | 883 | 5669 | 2834 | 6073 | 3037 |
| 4931 | 4930 | 5303 | 5304 | 5683 | 5684 | 6079 | 2026 |
| 4933 | 2467 | 5309 | 1327 | 5689 | 1422 | 6089 | 3044 |
| 4937 | 2469 | 5323 | 5324 | 5693 | 949 | 6091 | 870 |
| 4943 | 4944 | 5333 | 889 | 5701 | 475 | 6101 | 305 |
| 4951 | 4950 | 5347 | 5348 | 5711 | 5710 | 6113 | 3057 |
| 4957 | 2479 | 5351 | 5350 | 5717 | 2859 | 6121 | 3060 |
| 4967 | 4968 | 5381 | 269 | 5737 | 151 | 6131 | 6130 |
| 4969 | 92 | 5387 | 5388 | 5741 | 410 | 6133 | 3067 |
| 4973 | 2487 | 5393 | 2697 | 5743 | 5744 | 6143 | 6144 |
| 4987 | 4988 | 5399 | 5398 | 5749 | 1437 | 6151 | 6150 |
| 4993 | 2497 | 5407 | 5408 | 5779 | 54 | 6163 | 268 |
| 4999 | 4998 | 5413 | 2707 | 5783 | 5784 | 6173 | 3087 |
| 5003 | 5004 | 5417 | 129 | 5791 | 5790 | 6197 | 3099 |
| 5009 | 313 | 5419 | 5418 | 5801 | 580 | 6199 | 6198 |
| 5011 | 5010 | 5431 | 5430 | 5807 | 1936 | 6203 | 6204 |
| 5021 | 2510 | 5437 | 2719 | 5813 | 2907 | 6211 | 1242 |
| 5023 | 5024 | 5441 | 680 | 5821 | 485 | 6217 | 3109 |
| 5039 | 5038 | 5443 | 5444 | 5827 | 5828 | 6221 | 3110 |
| 5051 | 5050 | 5449 | 2724 | 5839 | 1946 | 6229 | 519 |
| 5059 | 1686 | 5471 | 5470 | 5843 | 1948 | 6247 | 6248 |
| 5077 | 2539 | 5477 | 2739 | 5849 | 731 | 6257 | 3129 |
| 5081 | 254 | 5479 | 5478 | 5851 | 5850 | 6263 | 216 |
| 5087 | 5088 | 5483 | 5484 | 5857 | 2929 | 6269 | 1567 |
| 5099 | 5098 | 5501 | 1375 | 5861 | 2930 | 6271 | 1254 |
| 5101 | 2550 | 5503 | 5504 | 5867 | 5868 | 6277 | 3139 |
| 5107 | 5108 | 5507 | 5508 | 5869 | 978 | 6287 | 6288 |
| 5113 | 2557 | 5519 | 5518 | 5879 | 5878 | 6299 | 6298 |
| 5119 | 5118 | 5521 | 2760 | 5881 | 196 | 6301 | 1575 |

# FIBONACCI ENTRY POINTS

| p | Z(p) | p | Z(p) | p | Z(p) | p | Z(p) |
|---|---|---|---|---|---|---|---|
| 6311 | 6310 | 6709 | 86 | 7109 | 1777 | 7541 | 3770 |
| 6317 | 3159 | 6719 | 6718 | 7121 | 3560 | 7547 | 7548 |
| 6323 | 6324 | 6733 | 481 | 7127 | 7128 | 7549 | 3774 |
| 6329 | 452 | 6737 | 1123 | 7129 | 3564 | 7559 | 7558 |
| 6337 | 3169 | 6761 | 845 | 7151 | 7150 | 7561 | 3780 |
| 6343 | 6344 | 6763 | 6764 | 7159 | 7158 | 7573 | 3787 |
| 6353 | 3177 | 6779 | 6778 | 7177 | 3589 | 7577 | 3789 |
| 6359 | 6358 | 6781 | 565 | 7187 | 2396 | 7583 | 7584 |
| 6361 | 1590 | 6791 | 6790 | 7193 | 3597 | 7589 | 3794 |
| 6367 | 6368 | 6793 | 3397 | 7207 | 7208 | 7591 | 7590 |
| 6373 | 3187 | 6803 | 6804 | 7211 | 1442 | 7603 | 7604 |
| 6379 | 6378 | 6823 | 6824 | 7213 | 3607 | 7607 | 7608 |
| 6389 | 3194 | 6827 | 6828 | 7219 | 2406 | 7621 | 1905 |
| 6397 | 457 | 6829 | 1707 | 7229 | 3614 | 7639 | 2546 |
| 6421 | 3210 | 6833 | 3417 | 7237 | 3619 | 7643 | 7644 |
| 6427 | 6428 | 6841 | 171 | 7243 | 7244 | 7649 | 478 |
| 6449 | 3224 | 6857 | 3429 | 7247 | 2416 | 7669 | 639 |
| 6451 | 6450 | 6863 | 6864 | 7253 | 3627 | 7673 | 3837 |
| 6469 | 539 | 6869 | 3434 | 7283 | 7284 | 7681 | 3840 |
| 6473 | 3237 | 6871 | 458 | 7297 | 3649 | 7687 | 7688 |
| 6481 | 1620 | 6883 | 6884 | 7307 | 812 | 7691 | 7690 |
| 6491 | 1298 | 6899 | 6898 | 7309 | 3654 | 7699 | 7698 |
| 6521 | 815 | 6907 | 6908 | 7321 | 1830 | 7703 | 7704 |
| 6529 | 1632 | 6911 | 6910 | 7331 | 7330 | 7717 | 3859 |
| 6547 | 6548 | 6917 | 1153 | 7333 | 3667 | 7723 | 7724 |
| 6551 | 6550 | 6947 | 6948 | 7349 | 3674 | 7727 | 7728 |
| 6553 | 3277 | 6949 | 3474 | 7351 | 7350 | 7741 | 1935 |
| 6563 | 2188 | 6959 | 6958 | 7369 | 921 | 7753 | 3877 |
| 6569 | 3284 | 6961 | 435 | 7393 | 3697 | 7757 | 1293 |
| 6571 | 6570 | 6967 | 6968 | 7411 | 7410 | 7759 | 7758 |
| 6577 | 3289 | 6971 | 6970 | 7417 | 3709 | 7789 | 1947 |
| 6581 | 470 | 6977 | 3489 | 7433 | 531 | 7793 | 3897 |
| 6599 | 6598 | 6983 | 776 | 7451 | 1490 | 7817 | 1303 |
| 6607 | 6608 | 6991 | 2330 | 7457 | 3729 | 7823 | 2608 |
| 6619 | 2206 | 6997 | 3499 | 7459 | 7458 | 7829 | 1957 |
| 6637 | 3319 | 7001 | 3500 | 7477 | 3739 | 7841 | 980 |
| 6653 | 3327 | 7013 | 1169 | 7481 | 3740 | 7853 | 1309 |
| 6659 | 6658 | 7019 | 7018 | 7487 | 7488 | 7867 | 1124 |
| 6661 | 3330 | 7027 | 1004 | 7489 | 416 | 7873 | 3937 |
| 6673 | 71 | 7039 | 7038 | 7499 | 7498 | 7877 | 3939 |
| 6679 | 742 | 7043 | 7044 | 7507 | 7508 | 7879 | 202 |
| 6689 | 3344 | 7057 | 3529 | 7517 | 1253 | 7883 | 876 |
| 6691 | 2230 | 7069 | 1178 | 7523 | 7524 | 7901 | 1975 |
| 6701 | 1675 | 7079 | 7078 | 7529 | 3764 | 7907 | 7908 |
| 6703 | 6704 | 7103 | 2368 | 7537 | 3769 | 7919 | 7918 |

# FIBONACCI ENTRY POINTS

| p | Z(p) | p | Z(p) | p | Z(p) | p | Z(p) |
|---|---|---|---|---|---|---|---|
| 7927 | 7928 | 8353 | 4177 | 8747 | 8748 | 9161 | 4580 |
| 7933 | 3967 | 8363 | 8364 | 8753 | 1459 | 9173 | 417 |
| 7937 | 1323 | 8369 | 523 | 8761 | 1460 | 9181 | 4590 |
| 7949 | 3974 | 8377 | 4189 | 8779 | 8778 | 9187 | 9188 |
| 7951 | 7950 | 8387 | 2796 | 8783 | 2928 | 9199 | 9198 |
| 7963 | 7964 | 8389 | 2097 | 8803 | 8804 | 9203 | 9204 |
| 7993 | 3997 | 8419 | 8418 | 8807 | 8808 | 9209 | 4604 |
| 8009 | 1001 | 8423 | 8424 | 8819 | 8818 | 9221 | 2305 |
| 8011 | 1602 | 8429 | 2107 | 8821 | 1470 | 9227 | 9228 |
| 8017 | 4009 | 8431 | 8430 | 8831 | 1766 | 9239 | 9238 |
| 8039 | 8038 | 8443 | 8444 | 8837 | 1473 | 9241 | 924 |
| 8053 | 4027 | 8447 | 8448 | 8839 | 8838 | 9257 | 4629 |
| 8059 | 2686 | 8461 | 470 | 8849 | 2212 | 9277 | 4639 |
| 8069 | 4034 | 8467 | 8468 | 8861 | 443 | 9281 | 4640 |
| 8081 | 1010 | 8501 | 2125 | 8863 | 8864 | 9283 | 844 |
| 8087 | 8088 | 8513 | 129 | 8867 | 8868 | 9293 | 4647 |
| 8089 | 1348 | 8521 | 4260 | 8887 | 8888 | 9311 | 9310 |
| 8093 | 1349 | 8527 | 8528 | 8893 | 4447 | 9319 | 3106 |
| 8101 | 4050 | 8537 | 4269 | 8923 | 8924 | 9323 | 3108 |
| 8111 | 8110 | 8539 | 8538 | 8929 | 1488 | 9337 | 667 |
| 8117 | 369 | 8543 | 2848 | 8933 | 4467 | 9341 | 2335 |
| 8123 | 2708 | 8563 | 8564 | 8941 | 4470 | 9343 | 9344 |
| 8147 | 8148 | 8573 | 1429 | 8951 | 8950 | 9349 | 38 |
| 8161 | 2040 | 8581 | 143 | 8963 | 8964 | 9371 | 1874 |
| 8167 | 8168 | 8597 | 4299 | 8969 | 118 | 9377 | 521 |
| 8171 | 1634 | 8599 | 2866 | 8971 | 2990 | 9391 | 9390 |
| 8179 | 8178 | 8609 | 4304 | 8999 | 8998 | 9397 | 4699 |
| 8191 | 8190 | 8623 | 8624 | 9001 | 375 | 9403 | 9404 |
| 8209 | 513 | 8627 | 8628 | 9007 | 9008 | 9413 | 4707 |
| 8219 | 8218 | 8629 | 2157 | 9011 | 1802 | 9419 | 9418 |
| 8221 | 4110 | 8641 | 360 | 9013 | 4507 | 9421 | 2355 |
| 8231 | 8230 | 8647 | 8648 | 9029 | 4514 | 9431 | 1886 |
| 8233 | 4117 | 8663 | 8664 | 9041 | 4520 | 9433 | 4717 |
| 8237 | 4119 | 8669 | 4334 | 9043 | 9044 | 9437 | 4719 |
| 8243 | 2748 | 8677 | 4339 | 9049 | 1508 | 9439 | 726 |
| 8263 | 8264 | 8681 | 2170 | 9059 | 9058 | 9461 | 2365 |
| 8269 | 2067 | 8689 | 181 | 9067 | 9068 | 9463 | 9464 |
| 8273 | 4137 | 8693 | 161 | 9091 | 1818 | 9467 | 9468 |
| 8287 | 8288 | 8699 | 8698 | 9103 | 9104 | 9473 | 4737 |
| 8291 | 8290 | 8707 | 8708 | 9109 | 4554 | 9479 | 9478 |
| 8293 | 4147 | 8713 | 4357 | 9127 | 9128 | 9491 | 9490 |
| 8297 | 4149 | 8719 | 8718 | 9133 | 4567 | 9497 | 1583 |
| 8311 | 8310 | 8731 | 2910 | 9137 | 4569 | 9511 | 9510 |
| 8317 | 4159 | 8737 | 4369 | 9151 | 3050 | 9521 | 85 |
| 8329 | 2082 | 8741 | 4370 | 9157 | 4579 | 9533 | 4767 |

## FIBONACCI ENTRY POINTS

| p | Z(p) |
|---|---|
| 9539 | 9538 |
| 9547 | 9548 |
| 9551 | 9550 |
| 9587 | 9588 |
| 9601 | 1600 |
| 9613 | 4807 |
| 9619 | 9618 |
| 9623 | 9624 |
| 9629 | 4814 |
| 9631 | 9630 |
| 9643 | 9644 |
| 9649 | 4824 |
| 9661 | 966 |
| 9677 | 1613 |
| 9679 | 3226 |
| 9689 | 2422 |
| 9697 | 373 |
| 9719 | 9718 |
| 9721 | 4860 |
| 9733 | 4867 |
| 9739 | 3246 |
| 9743 | 1392 |
| 9749 | 4874 |
| 9767 | 3256 |
| 9769 | 2442 |
| 9781 | 4890 |
| 9787 | 9788 |
| 9791 | 9790 |
| 9803 | 3268 |
| 9811 | 9810 |
| 9817 | 4909 |
| 9829 | 4914 |
| 9833 | 4917 |
| 9839 | 9838 |
| 9851 | 9850 |
| 9857 | 4929 |
| 9859 | 9858 |
| 9871 | 9870 |
| 9883 | 1412 |
| 9887 | 9888 |
| 9001 | 66 |
| 9907 | 9908 |
| 9923 | 3308 |
| 9929 | 1241 |
| 9931 | 1986 |

| p | Z(p) |
|---|---|
| 9941 | 994 |
| 9949 | 829 |
| 9967 | 9968 |
| 9973 | 4987 |

## ENTRY POINTS AND PERIODS FOR

## THE FIBONACCI SEQUENCE

The table shows the entry point (Z) and period (k) for all integers up to 1000.

| N | Z | k | N | Z | k | N | Z | k |
|---|---|---|---|---|---|---|---|---|
| | | | 41 | 20 | 40 | 81 | 108 | 216 |
| 2 | 3 | 3 | 42 | 24 | 48 | 82 | 60 | 120 |
| 3 | 4 | 8 | 43 | 44 | 88 | 83 | 84 | 168 |
| 4 | 6 | 6 | 44 | 30 | 30 | 84 | 24 | 48 |
| 5 | 5 | 20 | 45 | 60 | 120 | 85 | 45 | 180 |
| 6 | 12 | 24 | 46 | 24 | 48 | 86 | 132 | 264 |
| 7 | 8 | 16 | 47 | 16 | 32 | 87 | 28 | 56 |
| 8 | 6 | 12 | 48 | 12 | 24 | 88 | 30 | 60 |
| 9 | 12 | 24 | 49 | 56 | 112 | 89 | 11 | 44 |
| 10 | 15 | 60 | 50 | 75 | 300 | 90 | 60 | 120 |
| 11 | 10 | 10 | 51 | 36 | 72 | 91 | 56 | 112 |
| 12 | 12 | 24 | 52 | 42 | 84 | 92 | 24 | 48 |
| 13 | 7 | 28 | 53 | 27 | 108 | 93 | 60 | 120 |
| 14 | 24 | 48 | 54 | 36 | 72 | 94 | 48 | 96 |
| 15 | 20 | 40 | 55 | 10 | 20 | 95 | 90 | 180 |
| 16 | 12 | 24 | 56 | 24 | 48 | 96 | 24 | 48 |
| 17 | 9 | 36 | 57 | 36 | 72 | 97 | 49 | 196 |
| 18 | 12 | 24 | 58 | 42 | 42 | 98 | 168 | 336 |
| 19 | 18 | 18 | 59 | 58 | 58 | 99 | 60 | 120 |
| 20 | 30 | 60 | 60 | 60 | 120 | 100 | 150 | 300 |
| 21 | 8 | 16 | 61 | 15 | 60 | 101 | 50 | 50 |
| 22 | 30 | 30 | 62 | 30 | 30 | 102 | 36 | 72 |
| 23 | 24 | 48 | 63 | 24 | 48 | 103 | 104 | 208 |
| 24 | 12 | 24 | 64 | 48 | 96 | 104 | 42 | 84 |
| 25 | 25 | 100 | 65 | 35 | 140 | 105 | 40 | 80 |
| 26 | 21 | 84 | 66 | 60 | 120 | 106 | 27 | 108 |
| 27 | 36 | 72 | 67 | 68 | 136 | 107 | 36 | 72 |
| 28 | 24 | 48 | 68 | 18 | 36 | 108 | 36 | 72 |
| 29 | 14 | 14 | 69 | 24 | 48 | 109 | 27 | 108 |
| 30 | 60 | 120 | 70 | 120 | 240 | 110 | 30 | 60 |
| 31 | 30 | 30 | 71 | 70 | 70 | 111 | 76 | 152 |
| 32 | 24 | 48 | 72 | 12 | 24 | 112 | 24 | 48 |
| 33 | 20 | 40 | 73 | 37 | 148 | 113 | 19 | 76 |
| 34 | 9 | 36 | 74 | 57 | 228 | 114 | 36 | 72 |
| 35 | 40 | 80 | 75 | 100 | 200 | 115 | 120 | 240 |
| 36 | 12 | 24 | 76 | 18 | 18 | 116 | 42 | 42 |
| 37 | 19 | 76 | 77 | 40 | 80 | 117 | 84 | 168 |
| 38 | 18 | 18 | 78 | 84 | 168 | 118 | 174 | 174 |
| 39 | 28 | 56 | 79 | 78 | 78 | 119 | 72 | 144 |
| 40 | 30 | 60 | 80 | 60 | 120 | 120 | 60 | 120 |

ENTRY POINTS AND PERIODS FOR

THE FIBONACCI SEQUENCE

| N | Z | k | N | Z | k | N | Z | k |
|---|---|---|---|---|---|---|---|---|
| 121 | 110 | 110 | 161 | 24 | 48 | 201 | 68 | 136 |
| 122 | 15 | 60 | 162 | 108 | 216 | 202 | 150 | 150 |
| 123 | 20 | 40 | 163 | 164 | 328 | 203 | 56 | 112 |
| 124 | 30 | 30 | 164 | 60 | 120 | 204 | 36 | 72 |
| 125 | 125 | 500 | 165 | 20 | 40 | 205 | 20 | 40 |
| 126 | 24 | 48 | 166 | 84 | 168 | 206 | 312 | 624 |
| 127 | 128 | 256 | 167 | 168 | 336 | 207 | 24 | 48 |
| 128 | 96 | 192 | 168 | 24 | 48 | 208 | 84 | 168 |
| 129 | 44 | 88 | 169 | 91 | 364 | 209 | 90 | 90 |
| 130 | 105 | 420 | 170 | 45 | 180 | 210 | 120 | 240 |
| 131 | 130 | 130 | 171 | 36 | 72 | 211 | 42 | 42 |
| 132 | 60 | 120 | 172 | 132 | 264 | 212 | 54 | 108 |
| 133 | 72 | 144 | 173 | 87 | 348 | 213 | 140 | 280 |
| 134 | 204 | 408 | 174 | 84 | 168 | 214 | 36 | 72 |
| 135 | 180 | 360 | 175 | 200 | 400 | 215 | 220 | 440 |
| 136 | 18 | 36 | 176 | 60 | 120 | 216 | 36 | 72 |
| 137 | 69 | 276 | 177 | 116 | 232 | 217 | 120 | 240 |
| 138 | 24 | 48 | 178 | 33 | 132 | 218 | 27 | 108 |
| 139 | 46 | 46 | 179 | 178 | 178 | 219 | 148 | 296 |
| 140 | 120 | 240 | 180 | 60 | 120 | 220 | 30 | 60 |
| 141 | 16 | 32 | 181 | 90 | 90 | 221 | 63 | 252 |
| 142 | 210 | 210 | 182 | 168 | 336 | 222 | 228 | 456 |
| 143 | 70 | 140 | 183 | 60 | 120 | 223 | 224 | 448 |
| 144 | 12 | 24 | 184 | 24 | 48 | 224 | 24 | 48 |
| 145 | 70 | 140 | 185 | 95 | 380 | 225 | 300 | 600 |
| 146 | 111 | 444 | 186 | 60 | 120 | 226 | 57 | 228 |
| 147 | 56 | 112 | 187 | 90 | 180 | 227 | 228 | 456 |
| 148 | 114 | 228 | 188 | 48 | 96 | 228 | 36 | 72 |
| 149 | 37 | 148 | 189 | 72 | 144 | 229 | 114 | 114 |
| 150 | 300 | 600 | 190 | 90 | 180 | 230 | 120 | 240 |
| 151 | 50 | 50 | 191 | 190 | 190 | 231 | 40 | 80 |
| 152 | 18 | 36 | 192 | 48 | 96 | 232 | 42 | 84 |
| 153 | 36 | 72 | 193 | 97 | 388 | 233 | 13 | 52 |
| 154 | 120 | 240 | 194 | 147 | 588 | 234 | 84 | 168 |
| 155 | 30 | 60 | 195 | 140 | 280 | 235 | 80 | 160 |
| 156 | 84 | 168 | 196 | 168 | 336 | 236 | 174 | 174 |
| 157 | 79 | 316 | 197 | 99 | 396 | 237 | 156 | 312 |
| 158 | 78 | 78 | 198 | 60 | 120 | 238 | 72 | 144 |
| 159 | 108 | 216 | 199 | 22 | 22 | 239 | 238 | 238 |
| 160 | 120 | 240 | 200 | 150 | 300 | 240 | 60 | 120 |

ENTRY POINTS AND PERIODS FOR

THE FIBONACCI SEQUENCE

| N | Z | k | N | Z | k | N | Z | k |
|---|---|---|---|---|---|---|---|---|
| 241 | 120 | 240 | 281 | 28 | 56 | 321 | 36 | 72 |
| 242 | 330 | 330 | 282 | 48 | 96 | 322 | 24 | 48 |
| 243 | 324 | 648 | 283 | 284 | 568 | 323 | 18 | 36 |
| 244 | 30 | 60 | 284 | 210 | 210 | 324 | 108 | 216 |
| 245 | 280 | 560 | 285 | 180 | 360 | 325 | 175 | 700 |
| 246 | 60 | 120 | 286 | 210 | 420 | 326 | 492 | 984 |
| 247 | 126 | 252 | 287 | 40 | 80 | 327 | 108 | 216 |
| 248 | 30 | 60 | 288 | 24 | 48 | 328 | 60 | 120 |
| 249 | 84 | 168 | 289 | 153 | 612 | 329 | 16 | 32 |
| 250 | 375 | 1500 | 290 | 210 | 420 | 330 | 60 | 120 |
| 251 | 250 | 250 | 291 | 196 | 392 | 331 | 110 | 110 |
| 252 | 24 | 48 | 292 | 222 | 444 | 332 | 84 | 168 |
| 253 | 120 | 240 | 293 | 147 | 588 | 333 | 228 | 456 |
| 254 | 384 | 768 | 294 | 168 | 336 | 334 | 168 | 336 |
| 255 | 180 | 360 | 295 | 290 | 580 | 335 | 340 | 680 |
| 256 | 192 | 384 | 296 | 114 | 228 | 336 | 24 | 48 |
| 257 | 129 | 516 | 297 | 180 | 360 | 337 | 169 | 676 |
| 258 | 132 | 264 | 298 | 111 | 444 | 338 | 273 | 1092 |
| 259 | 152 | 304 | 299 | 168 | 336 | 339 | 76 | 152 |
| 260 | 210 | 420 | 300 | 300 | 600 | 340 | 90 | 180 |
| 261 | 84 | 168 | 301 | 88 | 176 | 341 | 30 | 30 |
| 262 | 390 | 390 | 302 | 150 | 150 | 342 | 36 | 72 |
| 263 | 88 | 176 | 303 | 100 | 200 | 343 | 392 | 784 |
| 264 | 60 | 120 | 304 | 36 | 72 | 344 | 132 | 264 |
| 265 | 135 | 540 | 305 | 15 | 60 | 345 | 120 | 240 |
| 266 | 72 | 144 | 306 | 36 | 72 | 346 | 87 | 348 |
| 267 | 44 | 88 | 307 | 44 | 88 | 347 | 116 | 232 |
| 268 | 204 | 408 | 308 | 120 | 240 | 348 | 84 | 168 |
| 269 | 67 | 268 | 309 | 104 | 208 | 349 | 174 | 174 |
| 270 | 180 | 360 | 310 | 30 | 60 | 350 | 600 | 1200 |
| 271 | 270 | 270 | 311 | 310 | 310 | 351 | 252 | 504 |
| 272 | 36 | 72 | 312 | 84 | 168 | 352 | 120 | 240 |
| 273 | 56 | 112 | 313 | 157 | 628 | 353 | 59 | 236 |
| 274 | 69 | 276 | 314 | 237 | 948 | 354 | 348 | 696 |
| 275 | 50 | 100 | 315 | 120 | 240 | 355 | 70 | 140 |
| 276 | 24 | 48 | 316 | 78 | 78 | 356 | 66 | 132 |
| 277 | 139 | 556 | 317 | 159 | 636 | 357 | 72 | 144 |
| 278 | 138 | 138 | 318 | 108 | 216 | 358 | 534 | 534 |
| 279 | 60 | 120 | 319 | 70 | 70 | 359 | 358 | 358 |
| 280 | 120 | 240 | 320 | 240 | 480 | 360 | 60 | 120 |

ENTRY POINTS AND PERIODS FOR

THE FIBONACCI SEQUENCE

| N | Z | k | N | Z | k | N | Z | k |
|---|---|---|---|---|---|---|---|---|
| 361 | 342 | 342 | 401 | 100 | 200 | 441 | 168 | 336 |
| 362 | 90 | 90 | 402 | 204 | 408 | 442 | 63 | 252 |
| 363 | 220 | 440 | 403 | 210 | 420 | 443 | 444 | 888 |
| 364 | 168 | 336 | 404 | 150 | 150 | 444 | 228 | 456 |
| 365 | 185 | 740 | 405 | 540 | 1080 | 445 | 55 | 220 |
| 366 | 60 | 120 | 406 | 168 | 336 | 446 | 672 | 1344 |
| 367 | 368 | 736 | 407 | 190 | 380 | 447 | 148 | 296 |
| 368 | 24 | 48 | 408 | 36 | 72 | 448 | 48 | 96 |
| 369 | 60 | 120 | 409 | 204 | 408 | 449 | 224 | 448 |
| 370 | 285 | 1140 | 410 | 60 | 120 | 450 | 300 | 600 |
| 371 | 216 | 432 | 411 | 276 | 552 | 451 | 20 | 40 |
| 372 | 60 | 120 | 412 | 312 | 624 | 452 | 114 | 228 |
| 373 | 187 | 748 | 413 | 232 | 464 | 453 | 100 | 200 |
| 374 | 90 | 180 | 414 | 24 | 48 | 454 | 228 | 456 |
| 375 | 500 | 1000 | 415 | 420 | 840 | 455 | 280 | 560 |
| 376 | 48 | 96 | 416 | 168 | 336 | 456 | 36 | 72 |
| 377 | 14 | 28 | 417 | 92 | 184 | 457 | 229 | 916 |
| 378 | 72 | 144 | 418 | 90 | 90 | 458 | 114 | 114 |
| 379 | 378 | 378 | 419 | 418 | 418 | 459 | 36 | 72 |
| 380 | 90 | 180 | 420 | 120 | 240 | 460 | 120 | 240 |
| 381 | 128 | 256 | 421 | 21 | 84 | 461 | 46 | 46 |
| 382 | 570 | 570 | 422 | 42 | 42 | 462 | 120 | 240 |
| 383 | 384 | 768 | 423 | 48 | 96 | 463 | 464 | 928 |
| 384 | 96 | 192 | 424 | 54 | 108 | 464 | 84 | 168 |
| 385 | 40 | 80 | 425 | 225 | 900 | 465 | 60 | 120 |
| 386 | 291 | 1164 | 426 | 420 | 840 | 466 | 39 | 156 |
| 387 | 132 | 264 | 427 | 120 | 240 | 467 | 468 | 936 |
| 388 | 294 | 588 | 428 | 36 | 72 | 468 | 84 | 168 |
| 389 | 97 | 388 | 429 | 140 | 280 | 469 | 136 | 272 |
| 390 | 420 | 840 | 430 | 660 | 1320 | 470 | 240 | 480 |
| 391 | 72 | 144 | 431 | 430 | 430 | 471 | 316 | 632 |
| 392 | 168 | 336 | 432 | 36 | 72 | 472 | 174 | 348 |
| 393 | 260 | 520 | 433 | 217 | 868 | 473 | 220 | 440 |
| 394 | 99 | 396 | 434 | 120 | 240 | 474 | 156 | 312 |
| 395 | 390 | 780 | 435 | 140 | 280 | 475 | 450 | 900 |
| 396 | 60 | 120 | 436 | 54 | 108 | 476 | 72 | 144 |
| 397 | 199 | 796 | 437 | 72 | 144 | 477 | 108 | 216 |
| 398 | 66 | 66 | 438 | 444 | 888 | 478 | 714 | 714 |
| 399 | 72 | 144 | 439 | 438 | 438 | 479 | 478 | 478 |
| 400 | 300 | 600 | 440 | 30 | 60 | 480 | 120 | 240 |

ENTRY POINTS AND PERIODS FOR

THE FIBONACCI SEQUENCE

| N | Z | k | N | Z | k | N | Z | k |
|---|---|---|---|---|---|---|---|---|
| 481 | 133 | 532 | 521 | 26 | 26 | 561 | 180 | 360 |
| 482 | 120 | 240 | 522 | 84 | 168 | 562 | 84 | 168 |
| 483 | 24 | 48 | 523 | 524 | 1048 | 563 | 188 | 376 |
| 484 | 330 | 330 | 524 | 390 | 390 | 564 | 48 | 96 |
| 485 | 245 | 980 | 525 | 200 | 400 | 565 | 95 | 380 |
| 486 | 324 | 648 | 526 | 264 | 528 | 566 | 852 | 1704 |
| 487 | 488 | 976 | 527 | 90 | 180 | 567 | 216 | 432 |
| 488 | 30 | 60 | 528 | 60 | 120 | 568 | 210 | 420 |
| 489 | 164 | 328 | 529 | 552 | 1104 | 569 | 284 | 568 |
| 490 | 840 | 1680 | 530 | 135 | 540 | 570 | 180 | 360 |
| 491 | 490 | 490 | 531 | 348 | 696 | 571 | 570 | 570 |
| 492 | 60 | 120 | 532 | 72 | 144 | 572 | 210 | 420 |
| 493 | 126 | 252 | 533 | 140 | 280 | 573 | 380 | 760 |
| 494 | 126 | 252 | 534 | 132 | 264 | 574 | 120 | 240 |
| 495 | 60 | 120 | 535 | 180 | 360 | 575 | 600 | 1200 |
| 496 | 60 | 120 | 536 | 204 | 408 | 576 | 48 | 96 |
| 497 | 280 | 560 | 537 | 356 | 712 | 577 | 289 | 1156 |
| 498 | 84 | 168 | 538 | 201 | 804 | 578 | 153 | 612 |
| 499 | 498 | 498 | 539 | 280 | 560 | 579 | 388 | 776 |
| 500 | 750 | 1500 | 540 | 180 | 360 | 580 | 210 | 420 |
| 501 | 168 | 336 | 541 | 90 | 90 | 581 | 168 | 336 |
| 502 | 750 | 750 | 542 | 270 | 270 | 582 | 588 | 1176 |
| 503 | 504 | 1008 | 543 | 180 | 360 | 583 | 270 | 540 |
| 504 | 24 | 48 | 544 | 72 | 144 | 584 | 222 | 444 |
| 505 | 50 | 100 | 545 | 135 | 540 | 585 | 420 | 840 |
| 506 | 120 | 240 | 546 | 168 | 336 | 586 | 147 | 588 |
| 507 | 364 | 728 | 547 | 548 | 1096 | 587 | 588 | 1176 |
| 508 | 384 | 768 | 548 | 138 | 276 | 588 | 168 | 336 |
| 509 | 254 | 254 | 549 | 60 | 120 | 589 | 90 | 90 |
| 510 | 180 | 360 | 550 | 150 | 300 | 590 | 870 | 1740 |
| 511 | 296 | 592 | 551 | 126 | 126 | 591 | 396 | 792 |
| 512 | 384 | 768 | 552 | 24 | 48 | 592 | 228 | 456 |
| 513 | 36 | 72 | 553 | 312 | 624 | 593 | 297 | 1188 |
| 514 | 129 | 516 | 554 | 417 | 1668 | 594 | 180 | 360 |
| 515 | 520 | 1040 | 555 | 380 | 760 | 595 | 360 | 720 |
| 516 | 132 | 264 | 556 | 138 | 138 | 596 | 222 | 444 |
| 517 | 80 | 160 | 557 | 31 | 124 | 597 | 44 | 88 |
| 518 | 456 | 912 | 558 | 60 | 120 | 598 | 168 | 336 |
| 519 | 348 | 696 | 559 | 308 | 616 | 599 | 598 | 598 |
| 520 | 210 | 420 | 560 | 120 | 240 | 600 | 300 | 600 |

ENTRY POINTS AND PERIODS FOR

THE FIBONACCI SEQUENCE

| N | Z | k | N | Z | k | N | Z | k |
|---|---|---|---|---|---|---|---|---|
| 601 | 300 | 600 | 641 | 320 | 640 | 681 | 228 | 456 |
| 602 | 264 | 528 | 642 | 36 | 72 | 682 | 30 | 30 |
| 603 | 204 | 408 | 643 | 644 | 1288 | 683 | 684 | 1368 |
| 604 | 150 | 150 | 644 | 24 | 48 | 684 | 36 | 72 |
| 605 | 110 | 220 | 645 | 220 | 440 | 685 | 345 | 1380 |
| 606 | 300 | 600 | 646 | 18 | 36 | 686 | 1176 | 2352 |
| 607 | 608 | 1216 | 647 | 648 | 1296 | 687 | 228 | 456 |
| 608 | 72 | 144 | 648 | 108 | 216 | 688 | 132 | 264 |
| 609 | 56 | 112 | 649 | 290 | 290 | 689 | 189 | 756 |
| 610 | 15 | 60 | 650 | 525 | 2100 | 690 | 120 | 240 |
| 611 | 112 | 224 | 651 | 120 | 240 | 691 | 138 | 138 |
| 612 | 36 | 72 | 652 | 492 | 984 | 692 | 174 | 348 |
| 613 | 307 | 1228 | 653 | 327 | 1308 | 693 | 120 | 240 |
| 614 | 132 | 264 | 654 | 108 | 216 | 694 | 348 | 696 |
| 615 | 20 | 40 | 655 | 130 | 260 | 695 | 230 | 460 |
| 616 | 120 | 240 | 656 | 60 | 120 | 696 | 84 | 168 |
| 617 | 309 | 1236 | 657 | 444 | 888 | 697 | 180 | 360 |
| 618 | 312 | 624 | 658 | 48 | 96 | 698 | 174 | 174 |
| 619 | 206 | 206 | 659 | 658 | 658 | 699 | 52 | 104 |
| 620 | 30 | 60 | 660 | 60 | 120 | 700 | 600 | 1200 |
| 621 | 72 | 144 | 661 | 55 | 220 | 701 | 175 | 700 |
| 622 | 930 | 930 | 662 | 330 | 330 | 702 | 252 | 504 |
| 623 | 88 | 176 | 663 | 252 | 504 | 703 | 342 | 684 |
| 624 | 84 | 168 | 664 | 84 | 168 | 704 | 240 | 480 |
| 625 | 625 | 2500 | 665 | 360 | 720 | 705 | 80 | 160 |
| 626 | 471 | 1884 | 666 | 228 | 456 | 706 | 177 | 708 |
| 627 | 180 | 360 | 667 | 168 | 336 | 707 | 200 | 400 |
| 628 | 474 | 948 | 668 | 168 | 336 | 708 | 174 | 348 |
| 629 | 171 | 684 | 669 | 224 | 448 | 709 | 118 | 118 |
| 630 | 120 | 240 | 670 | 1020 | 2040 | 710 | 210 | 420 |
| 631 | 630 | 630 | 671 | 30 | 60 | 711 | 156 | 312 |
| 632 | 78 | 156 | 672 | 24 | 48 | 712 | 66 | 132 |
| 633 | 84 | 168 | 673 | 337 | 1348 | 713 | 120 | 240 |
| 634 | 159 | 636 | 674 | 507 | 2028 | 714 | 72 | 144 |
| 635 | 640 | 1280 | 675 | 900 | 1800 | 715 | 70 | 140 |
| 636 | 108 | 216 | 676 | 546 | 1092 | 716 | 534 | 534 |
| 637 | 56 | 112 | 677 | 113 | 452 | 717 | 476 | 952 |
| 638 | 210 | 210 | 678 | 228 | 456 | 718 | 1074 | 1074 |
| 639 | 420 | 840 | 679 | 392 | 784 | 719 | 718 | 718 |
| 640 | 480 | 960 | 680 | 90 | 180 | 720 | 60 | 120 |

ENTRY POINTS AND PERIODS FOR

THE FIBONACCI SEQUENCE

| N | Z | k | N | Z | k | N | Z | k |
|---|---|---|---|---|---|---|---|---|
| 721 | 104 | 208 | 761 | 95 | 380 | 801 | 132 | 264 |
| 722 | 342 | 342 | 762 | 384 | 768 | 802 | 300 | 600 |
| 723 | 120 | 240 | 763 | 216 | 432 | 803 | 370 | 740 |
| 724 | 90 | 90 | 764 | 570 | 570 | 804 | 204 | 408 |
| 725 | 350 | 700 | 765 | 180 | 360 | 805 | 120 | 240 |
| 726 | 660 | 1320 | 766 | 384 | 768 | 806 | 210 | 420 |
| 727 | 728 | 1456 | 767 | 406 | 812 | 807 | 268 | 536 |
| 728 | 168 | 336 | 768 | 192 | 384 | 808 | 150 | 300 |
| 729 | 972 | 1944 | 769 | 96 | 192 | 809 | 202 | 202 |
| 730 | 555 | 2220 | 770 | 120 | 240 | 810 | 540 | 1080 |
| 731 | 396 | 792 | 771 | 516 | 1032 | 811 | 270 | 270 |
| 732 | 60 | 120 | 772 | 582 | 1164 | 812 | 168 | 336 |
| 733 | 367 | 1468 | 773 | 387 | 1548 | 813 | 540 | 1080 |
| 734 | 1104 | 2208 | 774 | 132 | 264 | 814 | 570 | 1140 |
| 735 | 280 | 560 | 775 | 150 | 300 | 815 | 820 | 1640 |
| 736 | 24 | 48 | 776 | 294 | 588 | 816 | 36 | 72 |
| 737 | 340 | 680 | 777 | 152 | 304 | 817 | 396 | 792 |
| 738 | 60 | 120 | 778 | 291 | 1164 | 818 | 204 | 408 |
| 739 | 738 | 738 | 779 | 180 | 360 | 819 | 168 | 336 |
| 740 | 570 | 1140 | 780 | 420 | 840 | 820 | 60 | 120 |
| 741 | 252 | 504 | 781 | 70 | 70 | 821 | 205 | 820 |
| 742 | 216 | 432 | 782 | 72 | 144 | 822 | 276 | 552 |
| 743 | 248 | 496 | 783 | 252 | 504 | 823 | 824 | 1648 |
| 744 | 60 | 120 | 784 | 168 | 336 | 824 | 312 | 624 |
| 745 | 185 | 740 | 785 | 395 | 1580 | 825 | 100 | 200 |
| 746 | 561 | 2244 | 786 | 780 | 1560 | 826 | 696 | 1392 |
| 747 | 84 | 168 | 787 | 788 | 1576 | 827 | 828 | 1656 |
| 748 | 90 | 180 | 788 | 198 | 396 | 828 | 24 | 48 |
| 749 | 72 | 144 | 789 | 88 | 176 | 829 | 69 | 276 |
| 750 | 1500 | 3000 | 790 | 390 | 780 | 830 | 420 | 840 |
| 751 | 750 | 750 | 791 | 152 | 304 | 831 | 417 | 1668 |
| 752 | 48 | 96 | 792 | 60 | 120 | 832 | 336 | 672 |
| 753 | 500 | 1000 | 793 | 105 | 420 | 833 | 504 | 1008 |
| 754 | 42 | 84 | 794 | 597 | 2388 | 834 | 276 | 552 |
| 755 | 50 | 100 | 795 | 540 | 1080 | 835 | 840 | 1680 |
| 756 | 72 | 144 | 796 | 66 | 66 | 836 | 90 | 90 |
| 757 | 379 | 1516 | 797 | 57 | 228 | 837 | 180 | 360 |
| 758 | 378 | 378 | 798 | 72 | 144 | 838 | 1254 | 1254 |
| 759 | 120 | 240 | 799 | 144 | 288 | 839 | 838 | 838 |
| 760 | 90 | 180 | 800 | 600 | 1200 | 840 | 120 | 240 |

ENTRY POINTS AND PERIODS FOR

THE FIBONACCI SEQUENCE

| N | Z | k | N | Z | k | N | Z | k |
|---|---|---|---|---|---|---|---|---|
| 841 | 406 | 406 | 881 | 88 | 176 | 921 | 44 | 88 |
| 842 | 21 | 84 | 882 | 168 | 336 | 922 | 138 | 138 |
| 843 | 28 | 56 | 883 | 884 | 1768 | 923 | 70 | 140 |
| 844 | 42 | 42 | 884 | 126 | 252 | 924 | 120 | 240 |
| 845 | 455 | 1820 | 885 | 580 | 1160 | 925 | 475 | 1900 |
| 846 | 48 | 96 | 886 | 444 | 888 | 926 | 1392 | 2784 |
| 847 | 440 | 880 | 887 | 888 | 1776 | 927 | 312 | 624 |
| 848 | 108 | 216 | 888 | 228 | 456 | 928 | 168 | 336 |
| 849 | 284 | 568 | 889 | 128 | 256 | 929 | 464 | 928 |
| 850 | 225 | 900 | 890 | 165 | 660 | 930 | 60 | 120 |
| 851 | 456 | 912 | 891 | 540 | 1080 | 931 | 72 | 144 |
| 852 | 420 | 840 | 892 | 672 | 1344 | 932 | 78 | 156 |
| 853 | 427 | 1708 | 893 | 144 | 288 | 933 | 620 | 1240 |
| 854 | 120 | 240 | 894 | 444 | 888 | 934 | 468 | 936 |
| 855 | 180 | 360 | 895 | 890 | 1780 | 935 | 90 | 180 |
| 856 | 36 | 72 | 896 | 96 | 192 | 936 | 84 | 168 |
| 857 | 429 | 1716 | 897 | 168 | 336 | 937 | 469 | 1876 |
| 858 | 420 | 840 | 898 | 672 | 1344 | 938 | 408 | 816 |
| 859 | 78 | 78 | 899 | 210 | 210 | 939 | 628 | 1256 |
| 860 | 660 | 1320 | 900 | 300 | 600 | 940 | 240 | 480 |
| 861 | 40 | 80 | 901 | 27 | 108 | 941 | 470 | 470 |
| 862 | 1290 | 1290 | 902 | 60 | 120 | 942 | 948 | 1896 |
| 863 | 864 | 1728 | 903 | 88 | 176 | 943 | 120 | 240 |
| 864 | 72 | 144 | 904 | 114 | 228 | 944 | 348 | 696 |
| 865 | 435 | 1740 | 905 | 90 | 180 | 945 | 360 | 720 |
| 866 | 651 | 2604 | 906 | 300 | 600 | 946 | 660 | 1320 |
| 867 | 612 | 1224 | 907 | 908 | 1816 | 947 | 948 | 1896 |
| 868 | 120 | 240 | 908 | 228 | 456 | 948 | 156 | 312 |
| 869 | 390 | 390 | 909 | 300 | 600 | 949 | 259 | 1036 |
| 870 | 420 | 840 | 910 | 840 | 1680 | 950 | 450 | 900 |
| 871 | 476 | 952 | 911 | 70 | 70 | 951 | 636 | 1272 |
| 872 | 54 | 108 | 912 | 36 | 72 | 952 | 72 | 144 |
| 873 | 588 | 1176 | 913 | 420 | 840 | 953 | 53 | 212 |
| 874 | 72 | 144 | 914 | 687 | 2748 | 954 | 108 | 216 |
| 875 | 1000 | 2000 | 915 | 60 | 120 | 955 | 190 | 380 |
| 876 | 444 | 888 | 916 | 114 | 114 | 956 | 714 | 714 |
| 877 | 439 | 1756 | 917 | 520 | 1040 | 957 | 140 | 280 |
| 878 | 438 | 438 | 918 | 36 | 72 | 958 | 1434 | 1434 |
| 879 | 588 | 1176 | 919 | 102 | 102 | 959 | 552 | 1104 |
| 880 | 60 | 120 | 920 | 120 | 240 | 960 | 240 | 480 |

ENTRY POINTS AND PERIODS FOR

THE FIBONACCI SEQUENCE

| N | Z | k |
|---|---|---|
| 961 | 839 | 839 |
| 962 | 399 | 1596 |
| 963 | 36 | 72 |
| 964 | 120 | 240 |
| 965 | 485 | 1940 |
| 966 | 24 | 48 |
| 967 | 88 | 176 |
| 968 | 330 | 660 |
| 969 | 36 | 72 |
| 970 | 735 | 2940 |
| 971 | 970 | 970 |
| 972 | 324 | 648 |
| 973 | 184 | 368 |
| 974 | 1464 | 2928 |
| 975 | 700 | 1400 |
| 976 | 60 | 120 |
| 977 | 163 | 652 |
| 978 | 492 | 984 |
| 979 | 110 | 440 |
| 980 | 840 | 1680 |
| 981 | 108 | 216 |
| 982 | 1470 | 1470 |
| 983 | 984 | 1968 |
| 984 | 60 | 120 |
| 985 | 495 | 1980 |
| 986 | 126 | 252 |
| 987 | 16 | 32 |
| 988 | 126 | 252 |
| 989 | 264 | 528 |
| 990 | 60 | 120 |
| 991 | 198 | 198 |
| 992 | 120 | 240 |
| 993 | 220 | 440 |
| 994 | 840 | 1680 |
| 995 | 110 | 220 |
| 996 | 84 | 168 |
| 997 | 499 | 1996 |
| 998 | 498 | 498 |
| 999 | 684 | 1368 |
| 1000 | 750 | 1500 |

## CHARACTERISTIC NUMBERS OF FIBONACCI SEQUENCES

The characteristic number (D) of a Fibonacci sequence ($T_n$) is the absolute value of the expression

$$T_n^2 - T_{n-1} T_{n+1}$$

for any n. (See FQ, Dec. 1963, pp. 43-46). The characteristic numbers are arranged in order of size up to 2000 together with the sequences to which the characteristic numbers belong. The pairs are conjugate in the sense that the absolute value of the alternating terms of each sequence corresponds to the positive portion of the other sequence. The factorizations exemplify the fact that every composite characteristic number is a product of other characteristic numbers.

| D | FACTORS | SEQUENCES |
|---|---|---|
| 1 | 1 | (0,1) |
| 5 | 5 | (1,3) |
| 11 | 11 | (1,4) (2,5) |
| 19 | 19 | (1,5) (3,7) |
| 29 | 29 | (1,6) (4,9) |
| 31 | 31 | (2,7) (3,8) |
| 41 | 41 | (1,7) (5,11) |
| 55 | 5*11 | (1,8) (6,13) |
| 59 | 59 | (2,9) (5,12) |
| 61 | 61 | (3,10) (4,11) |
| 71 | 71 | (1,9) (7,15) |
| 79 | 79 | (3,11) (5,13) |
| 89 | 89 | (1,10) (8,17) |
| 95 | 5*19 | (2,11) (7,16) |
| 101 | 101 | (4,13) (5,14) |
| 109 | 109 | (1,11) (9,19) |
| 121 | 11*11 | (3,13) (7,17) |
| 131 | 131 | (1,12) (10,21) |
| 139 | 139 | (2,13) (9,20) |
| 145 | 5*29 | (3,14) (8,19) |
| 149 | 149 | (4,15) (7,18) |
| 151 | 151 | (5,16) (6,17) |
| 155 | 5*31 | (1,13) (11,23) |
| 179 | 179 | (5,17) (7,19) |
| 181 | 181 | (1,14) (12,25) |
| 191 | 191 | (2,15) (11,24) |
| 199 | 199 | (3,16) (10,23) |
| 205 | 5*41 | (4,17) (9,22) |
| 209 | 11*19 | (1,15) (13,27) |
| | | (5,18) (8,21) |
| 211 | 211 | (6,19) (7,20) |
| 229 | 229 | (3,17) (11,25) |
| 239 | 239 | (1,16) (14,29) |
| 241 | 241 | (5,19) (9,23) |
| 251 | 251 | (2,17) (13,28) |
| 269 | 269 | (4,19) (11,26) |
| 271 | 271 | (1,17) (15,31) |
| 281 | 281 | (7,22) (8,23) |
| 295 | 5*59 | (3,19) (13,29) |
| 305 | 5*61 | (1,18) (16,33) |
| 311 | 311 | (5,21) (11,27) |
| 319 | 11*29 | (2,19) (15,32) |
| | | (7,23) (9,25) |
| 331 | 331 | (3,20) (14,31) |
| 341 | 11*31 | (1,19) (17,35) |
| | | (4,21) (13,30) |
| 349 | 349 | (5,22) (12,29) |
| 355 | 5*71 | (6,23) (11,28) |
| 359 | 359 | (7,24) (10,27) |
| 361 | 19*19 | (8,25) (9,26) |
| 379 | 379 | (1,20) (18,37) |
| 389 | 389 | (5,23) (13,31) |
| 395 | 5*79 | (2,21) (17,36) |
| 401 | 401 | (7,25) (11,29) |
| 409 | 409 | (3,22) (16,35) |
| 419 | 419 | (1,21) (19,39) |
| 421 | 421 | (4,23) (15,34) |
| 431 | 431 | (5,24) (14,33) |
| 439 | 439 | (6,25) (13,32) |
| 445 | 5*89 | (7,26) (12,31) |
| 449 | 449 | (8,27) (11,30) |
| 451 | 11*41 | (3,23) (17,37) |
| | | (9,28) (10,29) |
| 461 | 461 | (1,22) (20,41) |
| 479 | 479 | (2,23) (19,40) |
| 491 | 491 | (7,27) (13,33) |
| 499 | 499 | (9,29) (11,31) |
| 505 | 5*101 | (1,23) (21,43) |
| 509 | 509 | (4,25) (17,38) |
| 521 | 521 | (5,26) (16,37) |
| 541 | 541 | (3,25) (19,41) |
| 545 | 5*109 | (8,29) (13,34) |

## CHARACTERISTIC NUMBERS OF FIBONACCI SEQUENCES

| D | FACTORS | SEQUENCES |
|---|---|---|
| 551 | 19*29 | (1,24) (22,45) |
| | | (10,31) (11,32) |
| 569 | 569 | (5,27) (17,39) |
| 571 | 571 | (2,25) (21,44) |
| 589 | 19*31 | (3,26) (20,43) |
| | | (7,29) (15,37) |
| 599 | 599 | (1,25) (23,47) |
| 601 | 601 | (9,31) (13,35) |
| 605 | 5*11*11 | (4,27) (19,42) |
| 619 | 619 | (5,28) (18,41) |
| 631 | 631 | (6,29) (17,40) |
| 641 | 641 | (7,30) (16,39) |
| 649 | 11*59 | (1,26) (24,49) |
| | | (8,31) (15,38) |
| 655 | 5*131 | (9,32) (14,37) |
| 659 | 659 | (10,33) (13,36) |
| 661 | 661 | (11,34) (12,35) |
| 671 | 11*61 | (2,27) (23,48) |
| | | (5,29) (19,43) |
| 691 | 691 | (3,28) (22,47) |
| 695 | 5*139 | (7,31) (17,41) |
| 701 | 701 | (1,27) (25,51) |
| 709 | 709 | (4,29) (21,46) |
| 719 | 719 | (11,35) (13,37) |
| 739 | 739 | (6,31) (19,44) |
| 745 | 5*149 | (3,29) (23,49) |
| 751 | 751 | (7,32) (18,43) |
| 755 | 5*151 | (1,28) (26,53) |
| 761 | 761 | (8,33) (17,42) |
| 769 | 769 | (9,34) (16,41) |
| 779 | 19*41 | (2,29) (25,52) |
| | | (11,36) (14,39) |
| 781 | 11*71 | (5,31) (21,47) |
| | | (12,37) (13,38) |
| 809 | 809 | (7,33) (19,45) |
| 811 | 811 | (1,29) (27,55) |
| 821 | 821 | (4,31) (23,50) |
| 829 | 829 | (9,35) (17,43) |
| 839 | 839 | (5,32) (22,49) |
| 841 | 29*29 | (11,37) (15,41) |
| 859 | 859 | (3,31) (25,53) |
| 869 | 11*79 | (1,30) (28,57) |
| | | (7,34) (20,47) |
| 881 | 881 | (8,35) (19,46) |
| 895 | 5*179 | (2,31) (27,56) |
| 899 | 29*31 | (5,33) (23,51) |
| | | (10,37) (17,44) |
| 905 | 5*181 | (11,38) (16,43) |
| 911 | 911 | (13,40) (14,41) |
| 919 | 919 | (3,32) (26,55) |
| 929 | 929 | (1,31) (29,59) |
| 941 | 941 | (4,33) (25,54) |
| 955 | 5*191 | (9,37) (19,47) |
| 961 | 31*31 | (5,34) (24,53) |
| 971 | 971 | (11,39) (17,45) |
| 979 | 11*89 | (6,35) (23,52) |
| | | (13,41) (15,43) |
| 991 | 991 | (1,32) (30,61) |
| 995 | 5*199 | (7,36) (22,51) |
| 1009 | 1009 | (8,37) (21,50) |
| 1019 | 1019 | (2,33) (29,60) |
| 1021 | 1021 | (9,38) (20,49) |
| 1031 | 1031 | (10,39) (19,48) |
| 1039 | 1039 | (11,40) (18,47) |
| 1045 | 5*11*19 | (3,34) (28,59) |
| | | (12,41) (17,46) |
| 1049 | 1049 | (13,42) (16,45) |
| 1051 | 1051 | (14,43) (15,44) |
| 1055 | 5*211 | (1,33) (31,63) |
| 1061 | 1061 | (7,37) (23,53) |
| 1069 | 1069 | (4,35) (27,58) |
| 1091 | 1091 | (5,36) (26,57) |
| 1109 | 1109 | (11,41) (19,49) |
| 1111 | 11*101 | (3,35) (29,61) |
| | | (6,37) (25,56) |
| 1121 | 19*59 | (1,34) (32,65) |
| | | (13,43) (17,47) |
| 1129 | 1129 | (7,38) (24,55) |
| 1145 | 5*229 | (8,39) (23,54) |
| 1151 | 1151 | (2,35) (31,64) |
| 1159 | 19*61 | (5,37) (27,59) |
| | | (9,40) (22,53) |
| 1171 | 1171 | (10,41) (21,52) |
| 1181 | 1181 | (11,42) (20,51) |
| 1189 | 29*41 | (1,35) (33,67) |
| | | (12,43) (19,50) |
| 1195 | 5*239 | (13,44) (18,49) |
| 1199 | 11*109 | (7,39) (25,57) |
| | | (14,45) (17,48) |
| 1201 | 1201 | (15,46) (16,47) |
| 1205 | 5*241 | (4,37) (29,62) |
| 1229 | 1229 | (5,38) (28,61) |
| 1231 | 1231 | (9,41) (23,55) |
| 1249 | 1249 | (3,37) (31,65) |
| 1255 | 5*251 | (11,43) (21,53) |
| 1259 | 1259 | (1,36) (34,69) |

CHARACTERISTIC NUMBERS OF FIBONACCI SEQUENCES

| D | FACTORS | SEQUENCES |
|---|---|---|
| 1271 | 31*41 | (7,40) (26,59) |
| | | (13,45) (19,51) |
| 1279 | 1279 | (15,47) (17,49) |
| 1289 | 1289 | (8,41) (25,58) |
| 1291 | 1291 | (2,37) (33,68) |
| 1301 | 1301 | (5,39) (29,63) |
| 1319 | 1319 | (10,43) (23,56) |
| 1321 | 1321 | (3,38) (32,67) |
| 1331 | 11*11*11 | (1,37) (35,71) |
| 1345 | 5*269 | (7,41) (27,61) |
| 1349 | 19*71 | (4,39) (31,66) |
| | | (13,46) (20,53) |
| 1355 | 5*271 | (14,47) (19,52) |
| 1361 | 1361 | (16,49) (17,50) |
| 1381 | 1381 | (9,43) (25,59) |
| 1399 | 1399 | (6,41) (29,64) |
| 1405 | 5*281 | (1,38) (36,73) |
| 1409 | 1409 | (11,45) (23,57) |
| 1429 | 1429 | (13,47) (21,55) |
| 1439 | 1439 | (2,39) (35,72) |
| 1441 | 11*131 | (8,43) (27,62) |
| | | (15,49) (19,53) |
| 1451 | 1451 | (5,41) (31,67) |
| 1459 | 1459 | (9,44) (26,61) |
| 1471 | 1471 | (3,40) (34,71) |
| 1481 | 1481 | (1,39) (37,75) |
| 1489 | 1489 | (11,46) (24,59) |
| 1499 | 1499 | (7,43) (29,65) |
| 1501 | 19*79 | (4,41) (33,70) |
| | | (12,47) (23,58) |
| 1511 | 1511 | (13,48) (22,57) |
| 1529 | 11*139 | (5,42) (32,69) |
| | | (16,51) (19,54) |
| 1531 | 1531 | (17,52) (18,53) |
| 1549 | 1549 | (3,41) (35,73) |
| 1555 | 5*311 | (6,43) (31,68) |
| 1559 | 1559 | (1,40) (38,77) |
| 1571 | 1571 | (11,47) (25,61) |
| 1579 | 1579 | (7,44) (30,67) |
| 1595 | 5*11*29 | (2,41) (37,76) |
| | | (13,49) (23,59) |
| 1601 | 1601 | (8,45) (29,66) |
| 1609 | 1609 | (5,43) (33,71) |
| 1619 | 1619 | (17,53) (19,55) |
| 1621 | 1621 | (9,46) (28,65) |
| 1639 | 11*149 | (1,41) (39,79) |
| | | (10,47) (27,64) |
| 1655 | 5*331 | (11,48) (26,63) |

| D | FACTORS | SEQUENCES |
|---|---|---|
| 1661 | 11*151 | (4,43) (35,74) |
| | | (7,45) (31,69) |
| 1669 | 1669 | (25,62) (12,49) |
| 1681 | 41*41 | (13,50) (24,61) |
| 1691 | 19*89 | (5,44) (34,73) |
| | | (14,51) (23,60) |
| 1699 | 1699 | (15,52) (22,59) |
| 1705 | 5*11*31 | (9,47) (29,67) |
| | | (16,53) (21,58) |
| 1709 | 1709 | (17,54) (20,57) |
| 1711 | 29*59 | (3,43) (37,77) |
| | | (18,55) (19,56) |
| 1721 | 1721 | (1,42) (40,81) |
| 1741 | 1741 | (11,49) (27,65) |
| 1745 | 5*349 | (7,46) (32,71) |
| 1759 | 1759 | (2,43) (39,80) |
| 1769 | 29*61 | (8,47) (31,70) |
| | | (13,51) (25,63) |
| 1789 | 1789 | (15,53) (23,61) |
| 1795 | 5*359 | (3,44) (38,79) |
| 1801 | 1801 | (17,55) (21,59) |
| 1805 | 5*19*19 | (1,43) (41,83) |
| 1811 | 1811 | (10,49) (29,68) |
| 1829 | 31*59 | (4,45) (37,78) |
| | | (11,50) (28,67) |
| 1831 | 1831 | (7,47) (33,73) |
| 1861 | 1861 | (5,46) (36,77) |
| 1871 | 1871 | (14,53) (25,64) |
| 1879 | 1879 | (9,49) (31,71) |
| 1889 | 1889 | (16,55) (23,62) |
| 1891 | 31*61 | (1,44) (42,85) |
| | | (6,47) (35,76) |
| 1895 | 5*379 | (17,56) (22,61) |
| 1901 | 1901 | (19,58) (20,59) |
| 1919 | 19*101 | (7,48) (34,75) |
| | | (11,51) (29,69) |
| 1931 | 1931 | (2,45) (41,84) |
| 1945 | 5*389 | (8,49) (33,74) |
| 1949 | 1949 | (5,47) (37,79) |
| 1951 | 1951 | (13,53) (27,67) |
| 1969 | 11*179 | (3,46) (40,83) |
| | | (9,50) (32,73) |
| 1979 | 1979 | (1,45) (43,87) |
| 1991 | 11*181 | (10,51) (31,72) |
| | | (17,57) (23,63) |
| 1999 | 1999 | (19,59) (21,61) |

## SUMS OF FIBONACCI AND LUCAS RECIPROCALS

The value of $F_{400}$ is:

1760236806 4501396646 8226945392 4112507703 8438330449 2191886725
9928965753 4504421601 9675

The value of the reciprocal of $F_{400}$ is:

0.0000000000 0000000000 0000000000 0000000000 0000000000 0000000000
0000000000 0000000000 0005681053 8010320029 3322438451 0368971780
6515244969 6862102505 1895315230 1938174851 5725901070 2091114087
2874738506 2818531767 6846283034 8976183051 2241528312 4199639928
4572007688 5497059726 4269364270 2810904696 8678426926 6895612165
9731316887 2591747315 0111477837 9421208347 6329738124 6995989671
3002730598 1275547701 9213726740 9300242870

The value of the sum of the reciprocals of the Fibonacci numbers up to and including the reciprocal of $F_{400}$ is:

3.3598856662 4317755317 2011302918 9271796889 0513373196 8486495553
8153251303 1899668338 3606224078 3148035240 9760560946 4936311338
8688074279 5164523242 7845074278 2473755444 8608175851 9988130451
1796617849 5683596006 2293366068 2384275753 4146052269 3626746640
8256692967 5931016360 5230044858 6673777592 1417413466 2272028792
7516914824 8533820678 7957728691 3215675939 5985189161 9557398701
8322618333 5208484592 3683326483 0330158978

**********

The value of $L_{400}$ is:

3936009155 7196525704 7750929970 5107404603 7579426011 7590086228
3983953631 9776384862 8127

The value of the reciprocal of $L_{400}$ is:

0.0000000000 0000000000 0000000000 0000000000 0000000000 0000000000
0000000000 0000000000 0002540644 4965882246 9313101648 3493246429
9768877362 6597117385 5023600473 5099177173 0753562745 4687800042
0108353878 0278735485 5539200651 8754624189 4520207590 7322641134
6980097013 6341479824 3771481545 3578778954 3356958352 0433175245
9816595695 4768214285 8416845759 3441036358 9800738241 0567733357
7995400185 0675255004 7984769996 1960185993

The value of the sum of the reciprocals of the Lucas numbers up to and including the reciprocal of $L_{400}$ is:

1.9628581732 0964578286 8795128675 1835266495 9301716221 9421130715
2404170616 0754646037 7974931049 9352245824 1331645064 4057423987
2648364185 8468133707 5665470622 3278879254 7727222365 4808376912
7474193523 4783567143 9873344561 5152517654 1638682512 1369065746
6156751775 7831572512 3782472649 9046246089 2850046198 6608357696
0602438504 3256408629 9268257215 3538470202 5752437930 1424454072
5491825869 1310422266 8233338491 3885674017

# RESIDUE CYCLES OF FIBONACCI SEQUENCES

For any given modulus m, there are $m^2$ possible residue pairs which arrange themselves in cycles. The following tables account for all such residue pairs for the given moduli apart from the residue pair 0,0.

MODULUS 2

0 1 1

MODULUS 3

0 1 1 2 0 2 2 1

MODULUS 4

0 1 1 2 3 1

0 3 3 2 1 3

0 2 2

MODULUS 5

0 1 1 2 3 0 3 3 1 4 0 4 4 3 2 0 2 2 4 1

1 3 4 2

MODULUS 6

0 1 1 2 3 5 2 1 3 4 1 5 0 5 5 4 3 1 4 5

3 2 5 1

0 2 2 4 0 4 4 2

0 3 3

MODULUS 7

0 1 1 2 3 5 1 6 0 6 6 5 4 2 6 1

0 2 2 4 6 3 2 5 0 5 5 3 1 4 5 2

0 3 3 6 2 1 3 4 0 4 4 1 5 6 4 3

MODULUS 8

0 1 1 2 3 5 0 5 5 2 7 1

1 3 4 7 3 2 5 7 4 3 7 2

1 4 5 1 6 7 5 4 1 5 6 3

3 3 6 1 7 0 7 7 6 5 3 0

2 2 4 6 2 0

6 6 4 2 6 0

4 4 0

MODULUS 9

1 1 2 3 5 8 4 3 7 1 8 0 8 8 7 6 4 1 5 6

2 8 1 0

2 2 4 6 1 7 8 6 5 2 7 0 7 7 5 3 8 2 1 3

4 7 2 0

4 4 8 3 2 5 7 3 1 4 5 0 5 5 1 6 7 4 2 6

8 5 4 0

3 3 6 0 6 6 3 0

## RESIDUE CYCLES OF FIBONACCI SEQUENCES

MODULUS 10

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 8 | 3 | 1 | 4 | 5 | 9 | 4 | 3 | 7 | 0 | 7 | 7 | 4 | 1 | 5 |
| 6 | 1 | 7 | 8 | 5 | 3 | 8 | 1 | 9 | 0 | 9 | 9 | 8 | 7 | 5 | 2 | 7 | 9 | 6 | 5 |
| 1 | 6 | 7 | 3 | 0 | 3 | 3 | 6 | 9 | 5 | 4 | 9 | 3 | 2 | 5 | 7 | 2 | 9 | 1 | 0 |
| 2 | 2 | 4 | 6 | 0 | 6 | 6 | 2 | 8 | 0 | 8 | 8 | 6 | 4 | 0 | 4 | 4 | 8 | 2 | 0 |
| 1 | 3 | 4 | 7 | 1 | 8 | 9 | 7 | 6 | 3 | 9 | 2 | | | | | | | | |
| 2 | 6 | 8 | 4 | | | | | | | | | | | | | | | | |
| 5 | 5 | 0 | | | | | | | | | | | | | | | | | |

MODULUS 11

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 8 | 2 | 10 | 1 | 0 |
| 2 | 2 | 4 | 6 | 10 | 5 | 4 | 9 | 2 | 0 |
| 3 | 3 | 6 | 9 | 4 | 2 | 6 | 8 | 3 | 0 |
| 4 | 4 | 8 | 1 | 9 | 10 | 8 | 7 | 4 | 0 |
| 5 | 5 | 10 | 4 | 3 | 7 | 10 | 6 | 5 | 0 |
| 6 | 6 | 1 | 7 | 8 | 4 | 1 | 5 | 6 | 0 |
| 7 | 7 | 3 | 10 | 2 | 1 | 3 | 4 | 7 | 0 |
| 8 | 8 | 5 | 2 | 7 | 9 | 5 | 3 | 8 | 0 |
| 9 | 9 | 7 | 5 | 1 | 6 | 7 | 2 | 9 | 0 |
| 10 | 10 | 9 | 8 | 6 | 3 | 9 | 1 | 10 | 0 |
| 1 | 4 | 5 | 9 | 3 | | | | | |
| 2 | 8 | 10 | 7 | 6 | | | | | |
| 2 | 5 | 7 | 1 | 8 | 9 | 6 | 4 | 10 | 3 |

MODULUS 12

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 8 | 1 | 9 | 10 | 7 | 5 | 0 | 5 | 5 | 10 | 3 | 1 | 4 | 5 | 9 |
| 2 | 11 | 1 | 0 | | | | | | | | | | | | | | | | |
| 2 | 2 | 4 | 6 | 10 | 4 | 2 | 6 | 8 | 2 | 10 | 0 | 10 | 10 | 8 | 6 | 2 | 8 | 10 | 6 |
| 4 | 10 | 2 | 0 | | | | | | | | | | | | | | | | |
| 7 | 7 | 2 | 9 | 11 | 8 | 7 | 3 | 10 | 1 | 11 | 0 | 11 | 11 | 10 | 9 | 7 | 4 | 11 | 3 |
| 2 | 5 | 7 | 0 | | | | | | | | | | | | | | | | |
| 1 | 3 | 4 | 7 | 11 | 6 | 5 | 11 | 4 | 3 | 7 | 10 | 5 | 3 | 8 | 11 | 7 | 6 | 1 | 7 |
| 8 | 3 | 11 | 2 | | | | | | | | | | | | | | | | |
| 1 | 5 | 6 | 11 | 5 | 4 | 9 | 1 | 10 | 11 | 9 | 8 | 5 | 1 | 6 | 7 | 1 | 8 | 9 | 5 |
| 2 | 7 | 9 | 4 | | | | | | | | | | | | | | | | |
| 4 | 4 | 8 | 0 | 8 | 8 | 4 | 0 | | | | | | | | | | | | |
| 3 | 3 | 6 | 9 | 3 | 0 | | | | | | | | | | | | | | |
| 9 | 9 | 6 | 3 | 9 | 0 | | | | | | | | | | | | | | |
| 6 | 6 | 0 | | | | | | | | | | | | | | | | | |

## RESIDUE CYCLES OF FIBONACCI SEQUENCES

### MODULUS 13

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 8 | 0 | 8 | 8 | 3 | 11 | 1 | 12 | 0 | 12 | 12 | 11 | 10 | 8 | 5 |
| 0 | 5 | 5 | 10 | 2 | 12 | 1 | 0 | | | | | | | | | | | | |
| 2 | 2 | 4 | 6 | 10 | 3 | 0 | 3 | 3 | 6 | 9 | 2 | 11 | 0 | 11 | 11 | 9 | 7 | 3 | 10 |
| 0 | 10 | 10 | 7 | 4 | 11 | 2 | 0 | | | | | | | | | | | | |
| 4 | 4 | 8 | 12 | 7 | 6 | 0 | 6 | 6 | 12 | 5 | 4 | 9 | 0 | 9 | 9 | 5 | 1 | 6 | 7 |
| 0 | 7 | 7 | 1 | 8 | 9 | 4 | 0 | | | | | | | | | | | | |
| 1 | 3 | 4 | 7 | 11 | 5 | 3 | 8 | 11 | 6 | 4 | 10 | 1 | 11 | 12 | 10 | 9 | 6 | 2 | 8 |
| 10 | 5 | 2 | 7 | 9 | 3 | 12 | 2 | | | | | | | | | | | | |
| 1 | 5 | 6 | 11 | 4 | 2 | 6 | 8 | 1 | 9 | 10 | 6 | 3 | 9 | 12 | 8 | 7 | 2 | 9 | 11 |
| 7 | 5 | 12 | 4 | 3 | 7 | 10 | 4 | | | | | | | | | | | | |
| 1 | 4 | 5 | 9 | 1 | 10 | 11 | 8 | 6 | 1 | 7 | 8 | 2 | 10 | 12 | 9 | 8 | 4 | 12 | 3 |
| 2 | 5 | 7 | 12 | 6 | 5 | 11 | 3 | | | | | | | | | | | | |

### MODULUS 14

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 8 | 13 | 7 | 6 | 13 | 5 | 4 | 9 | 13 | 8 | 7 | 1 | 8 | 9 | 3 |
| 12 | 1 | 13 | 0 | 13 | 13 | 12 | 11 | 9 | 6 | 1 | 7 | 8 | 1 | 9 | 10 | 5 | 1 | 6 | 7 |
| 13 | 6 | 5 | 11 | 2 | 13 | 1 | 0 | | | | | | | | | | | | |
| 3 | 3 | 6 | 9 | 1 | 10 | 11 | 7 | 4 | 11 | 1 | 12 | 13 | 11 | 10 | 7 | 3 | 10 | 13 | 9 |
| 8 | 3 | 11 | 0 | 11 | 11 | 8 | 5 | 13 | 4 | 3 | 7 | 10 | 3 | 13 | 2 | 1 | 3 | 4 | 7 |
| 11 | 4 | 1 | 5 | 6 | 11 | 3 | 0 | | | | | | | | | | | | |
| 5 | 5 | 10 | 1 | 11 | 12 | 9 | 7 | 2 | 9 | 11 | 6 | 3 | 9 | 12 | 7 | 5 | 12 | 3 | 1 |
| 4 | 5 | 9 | 0 | 9 | 9 | 4 | 13 | 3 | 2 | 5 | 7 | 12 | 5 | 3 | 8 | 11 | 5 | 2 | 7 |
| 9 | 2 | 11 | 13 | 10 | 9 | 5 | 0 | | | | | | | | | | | | |
| 2 | 2 | 4 | 6 | 10 | 2 | 12 | 0 | 12 | 12 | 10 | 8 | 4 | 12 | 2 | 0 | | | | |
| 4 | 4 | 8 | 12 | 6 | 4 | 10 | 0 | 10 | 10 | 6 | 2 | 8 | 10 | 4 | 0 | | | | |
| 6 | 6 | 12 | 4 | 2 | 6 | 8 | 0 | 8 | 8 | 2 | 10 | 12 | 8 | 6 | 0 | | | | |
| 7 | 7 | 0 | | | | | | | | | | | | | | | | | |

### MODULUS 15

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 8 | 13 | 6 | 4 | 10 | 14 | 9 | 8 | 2 | 10 | 12 | 7 | 4 | 11 | 0 |
| 11 | 11 | 7 | 3 | 10 | 13 | 8 | 6 | 14 | 5 | 4 | 9 | 13 | 7 | 5 | 12 | 2 | 14 | 1 | 0 |
| 2 | 2 | 4 | 6 | 10 | 1 | 11 | 12 | 8 | 5 | 13 | 3 | 1 | 4 | 5 | 9 | 14 | 8 | 7 | 0 |
| 7 | 7 | 14 | 6 | 5 | 11 | 1 | 12 | 13 | 10 | 8 | 3 | 11 | 14 | 10 | 9 | 4 | 13 | 2 | 0 |
| 4 | 4 | 8 | 12 | 5 | 2 | 7 | 9 | 1 | 10 | 11 | 6 | 2 | 8 | 10 | 3 | 13 | 1 | 14 | 0 |
| 14 | 14 | 13 | 12 | 10 | 7 | 2 | 9 | 11 | 5 | 1 | 6 | 7 | 13 | 5 | 3 | 8 | 11 | 4 | 0 |
| 8 | 8 | 1 | 9 | 10 | 4 | 14 | 3 | 2 | 5 | 7 | 12 | 4 | 1 | 5 | 6 | 11 | 2 | 13 | 0 |
| 13 | 13 | 11 | 9 | 5 | 14 | 4 | 3 | 7 | 10 | 2 | 12 | 14 | 11 | 10 | 6 | 1 | 7 | 8 | 0 |
| 3 | 3 | 6 | 9 | 0 | 9 | 9 | 3 | 12 | 0 | 12 | 12 | 9 | 6 | 0 | 6 | 6 | 12 | 3 | 0 |
| 1 | 13 | 14 | 12 | 11 | 8 | 4 | 12 | | | | | | | | | | | | |
| 1 | 8 | 9 | 2 | 11 | 13 | 9 | 7 | | | | | | | | | | | | |
| 5 | 5 | 10 | 0 | 10 | 10 | 5 | 0 | | | | | | | | | | | | |

MODULUS 15 (continued)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 6 | 8 | 14 | 7 | 6 | 13 | 4 |
| 1 | 3 | 4 | 7 | 11 | 3 | 14 | 2 |
| 3 | 9 | 12 | 6 | | | | |

MODULUS 16

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 8 | 13 | 5 | 2 | 7 | 9 | 0 | 9 | 9 | 2 | 11 | 13 | 8 | 5 | 13 |
| 2 | 15 | 1 | 0 | | | | | | | | | | | | | | | | |
| 3 | 3 | 6 | 9 | 15 | 8 | 7 | 15 | 6 | 5 | 11 | 0 | 11 | 11 | 6 | 1 | 7 | 8 | 15 | 7 |
| 6 | 13 | 3 | 0 | | | | | | | | | | | | | | | | |
| 5 | 5 | 10 | 15 | 9 | 8 | 1 | 9 | 10 | 3 | 13 | 0 | 13 | 13 | 10 | 7 | 1 | 8 | 9 | 1 |
| 10 | 11 | 5 | 0 | | | | | | | | | | | | | | | | |
| 7 | 7 | 14 | 5 | 3 | 8 | 11 | 3 | 14 | 1 | 15 | 0 | 15 | 15 | 14 | 13 | 11 | 8 | 3 | 11 |
| 14 | 9 | 7 | 0 | | | | | | | | | | | | | | | | |
| 1 | 3 | 4 | 7 | 11 | 2 | 13 | 15 | 12 | 11 | 7 | 2 | 9 | 11 | 4 | 15 | 3 | 2 | 5 | 7 |
| 12 | 3 | 15 | 2 | | | | | | | | | | | | | | | | |
| 1 | 4 | 5 | 9 | 14 | 7 | 5 | 12 | 1 | 13 | 14 | 11 | 9 | 4 | 13 | 1 | 14 | 15 | 13 | 12 |
| 9 | 5 | 14 | 3 | | | | | | | | | | | | | | | | |
| 1 | 5 | 6 | 11 | 1 | 12 | 13 | 9 | 6 | 15 | 5 | 4 | 9 | 13 | 6 | 3 | 9 | 12 | 5 | 1 |
| 6 | 7 | 13 | 4 | | | | | | | | | | | | | | | | |
| 1 | 11 | 12 | 7 | 3 | 10 | 13 | 7 | 4 | 11 | 15 | 10 | 9 | 3 | 12 | 15 | 11 | 10 | 5 | 15 |
| 4 | 3 | 7 | 10 | | | | | | | | | | | | | | | | |
| 2 | 2 | 4 | 6 | 10 | 0 | 10 | 10 | 4 | 14 | 2 | 0 | | | | | | | | |
| 6 | 6 | 12 | 2 | 14 | 0 | 14 | 14 | 12 | 10 | 6 | 0 | | | | | | | | |
| 2 | 6 | 8 | 14 | 6 | 4 | 10 | 14 | 8 | 6 | 14 | 4 | | | | | | | | |
| 2 | 10 | 12 | 6 | 2 | 8 | 10 | 2 | 12 | 14 | 10 | 8 | | | | | | | | |
| 4 | 4 | 8 | 12 | 4 | 0 | | | | | | | | | | | | | | |
| 12 | 12 | 8 | 4 | 12 | 0 | | | | | | | | | | | | | | |
| 8 | 8 | 0 | | | | | | | | | | | | | | | | | |

MODULUS 17

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 8 | 13 | 4 | 0 | 4 | 4 | 8 | 12 | 3 | 15 | 1 | 16 | 0 | 16 | 16 |
| 15 | 14 | 12 | 9 | 4 | 13 | 0 | 13 | 13 | 9 | 5 | 14 | 2 | 16 | 1 | 0 | | | | |
| 2 | 2 | 4 | 6 | 10 | 16 | 9 | 8 | 0 | 8 | 8 | 16 | 7 | 6 | 13 | 2 | 15 | 0 | 15 | 15 |
| 13 | 11 | 7 | 1 | 8 | 9 | 0 | 9 | 9 | 1 | 10 | 11 | 4 | 15 | 2 | 0 | | | | |
| 3 | 3 | 6 | 9 | 15 | 7 | 5 | 12 | 0 | 12 | 12 | 7 | 2 | 9 | 11 | 3 | 14 | 0 | 14 | 14 |
| 11 | 8 | 2 | 10 | 12 | 5 | 0 | 5 | 5 | 10 | 15 | 8 | 6 | 14 | 3 | 0 | | | | |
| 6 | 6 | 12 | 1 | 13 | 14 | 10 | 7 | 0 | 7 | 7 | 14 | 4 | 1 | 5 | 6 | 11 | 0 | 11 | 11 |
| 5 | 16 | 4 | 3 | 7 | 10 | 0 | 10 | 10 | 3 | 13 | 16 | 12 | 11 | 6 | 0 | | | | |
| 1 | 3 | 4 | 7 | 11 | 1 | 12 | 13 | 8 | 4 | 12 | 16 | 11 | 10 | 4 | 14 | 1 | 15 | 16 | 14 |
| 13 | 10 | 6 | 16 | 5 | 4 | 9 | 13 | 5 | 1 | 6 | 7 | 13 | 3 | 16 | 2 | | | | |

MODULUS 17 (continued)

1 4 5 9 14 6 3 9 12 4 16 3 2 5 7 12 2 14 16 13
12 8 3 11 14 8 5 13 1 14 15 12 10 5 15 3

1 7 8 15 6 4 10 14 7 4 11 15 9 7 16 6 5 11 16 10
9 2 11 13 7 3 10 13 6 2 8 10 1 11 12 6

1 9 10 2 12 14 9 6 15 4 2 6 8 14 5 2 7 9 16 8
7 15 5 3 8 11 2 13 15 11 9 3 12 15 10 8

MODULUS 18

1 1 2 3 5 8 13 3 16 1 17 0 17 17 16 15 13 10 5 15
2 17 1 0

2 2 4 6 10 16 8 6 14 2 16 0 16 16 14 12 8 2 10 12
4 16 2 0

3 3 6 9 15 6 3 9 12 3 15 0 15 15 12 9 3 12 15 9
6 15 3 0

4 4 8 12 2 14 16 12 10 4 14 0 14 14 10 6 16 4 2 6
8 14 4 0

5 5 10 15 7 4 11 15 8 5 13 0 13 13 8 3 11 14 7 3
10 13 5 0

7 7 14 3 17 2 1 3 4 7 11 0 11 11 4 15 1 16 17 15
14 11 7 0

8 8 16 6 4 10 14 6 2 8 10 0 10 10 2 12 14 8 4 12
16 10 8 0

1 4 5 9 14 5 1 6 7 13 2 15 17 14 13 9 4 13 17 12
11 5 16 3

1 5 6 11 17 10 9 1 10 11 3 14 17 13 12 7 1 8 9 17
8 7 15 4

1 7 8 15 5 2 7 9 16 7 5 12 17 11 10 3 13 16 11 9
2 11 13 6

1 9 10 1 11 12 5 17 4 3 7 10 17 9 8 17 7 6 13 1
14 15 11 8

1 12 13 7 2 9 11 2 13 15 10 7 17 6 5 11 16 9 7 16
5 3 8 11

1 13 14 9 5 14 1 15 16 13 11 6 17 5 4 9 13 4 17 3
2 5 7 12

6 6 12 0 12 12 6 0

9 9 0

## ASYMPTOTIC RATIOS OF TERMS OF

## LINEAR RECURSION RELATIONS

For a linear recursion relation $T_{n+1} = a_1T_n + a_2T_{n-1} + \ldots\ldots + a_rT_{n-r+1}$ the limiting ratio of $T_{n+1}/T_n$ as n goes to infinity should such a limit exist is given by the root of greatest absolute value of the corresponding characteristic equation. Recursion relations are identified in the table by listing their coefficients $(a_1, a_2, \ldots\ldots, a_r)$.

| COEFFICIENTS | LIMITING RATIO |
|---|---|
| (1,1) | 1.61803399 |
| (1,1,1) | 1.83928676 |
| (1,1,1,1) | 1.92756198 |
| (1,1,1,1,1) | 1.96594824 |
| (1,1,1,1,1,1) | 1.98358284 |
| (1,1,1,1,1,1,1) | 1.99196420 |
| (1,1,1,1,1,1,1,1) | 1.99603118 |
| (1,1,1,1,1,1,1,1,1) | 1.99802947 |

| COEFFICIENTS | LIMITING RATIO |
|---|---|
| (1,1) | 1.61803399 |
| (2,1) | 2.41421356 |
| (3,1) | 3.30277564 |
| (4,1) | 4.23606798 |
| (5,1) | 5.19258240 |
| (6,1) | 6.16227766 |
| (7,1) | 7,14005494 |
| (8.1) | 8.12310563 |
| (9,1) | 9.10977223 |
| (10,1) | 10.09901951 |
| (11,1) | 11.09016994 |
| (12,1) | 12.08276253 |
| (13,1) | 13.07647322 |
| (14,1) | 14.07106781 |
| (15,1) | 15.06637298 |

| COEFFICIENTS | LIMITING RATIO |
|---|---|
| (3,-1) | 2.61803399 |
| (4,-1) | 3.73205081 |
| (5,-1) | 4.79128785 |
| (6,-1) | 5.82842712 |
| (7,-1) | 6.85410197 |
| (8,-1) | 7.87298335 |
| (9,-1) | 8.88748219 |
| (10,-1) | 9.89897949 |
| (11,-1) | 10.90832691 |
| (12,-1) | 11.91607978 |
| (13,-1) | 12.92261629 |
| (14,-1) | 13.92820323 |
| (15,-1) | 14.93303437 |

## FACTORIZATIONS OF THE TERMS OF THE FIBONACCI SEQUENCE WITH $T_1 = 2$, $T_2 = 5$

| n | TERM | FACTORIZATION |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 5 | 5 |
| 3 | 7 | 7 |
| 4 | 12 | 2*2*3 |
| 5 | 19 | 19 |
| 6 | 31 | 31 |
| 7 | 50 | 2*5*5 |
| 8 | 81 | 3*3*3*3 |
| 9 | 131 | 131 |
| 10 | 212 | 2*2*53 |
| 11 | 343 | 7*7*7 |
| 12 | 555 | 3*5*37 |
| 13 | 898 | 2*449 |
| 14 | 1453 | 1453 |
| 15 | 2351 | 2351 |
| 16 | 3804 | 2*2*3*317 |
| 17 | 6155 | 5*1231 |
| 18 | 9959 | 23*433 |
| 19 | 16114 | 2*7*1151 |
| 20 | 26073 | 3*3*2897 |
| 21 | 42187 | 42187 |
| 22 | 68260 | 2*2*5*3413 |
| 23 | 110447 | 19*5813 |
| 24 | 178707 | 3*71*839 |
| 25 | 289154 | 2*144577 |
| 26 | 467861 | 67*6983 |
| 27 | 757015 | 5*7*43*503 |
| 28 | 1224876 | 2*2*3*103*991 |
| 29 | 1981891 | 1981891 |
| 30 | 3206767 | 3206767 |
| 31 | 5188658 | 2*37*70117 |
| 32 | 8395425 | 3*3*5*5*37313 |
| 33 | 13584083 | 13584083 |
| 34 | 21979508 | 2*2*397*13841 |
| 35 | 35563591 | 7*83*61211 |
| 36 | 57543099 | 3*31*401*1543 |
| 37 | 93106690 | 2*5*53*175673 |
| 38 | 150649789 | 6257*24077 |
| 39 | 243756479 | 919*265241 |
| 40 | 394406268 | 2*2*3*59*97*5743 |
| 41 | 638162747 | 19*33587513 |
| 42 | 1032569015 | 5*23*229*39209 |
| 43 | 1670731762 | 2*7*2677*44579 |
| 44 | 2703300777 | 3*3*3*599*167149 |
| 45 | 4374032539 | 2693*1624223 |

FACTORIZATIONS OF THE TERMS OF THE

FIBONACCI SEQUENCE WITH $T_1 = 1$, $T_2 = 4$

| n | TERM | FACTORIZATION |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 4 | 2*2 |
| 3 | 5 | 5 |
| 4 | 9 | 3*3 |
| 5 | 14 | 2*7 |
| 6 | 23 | 23 |
| 7 | 37 | 37 |
| 8 | 60 | 2*2*3*5 |
| 9 | 97 | 97 |
| 10 | 157 | 157 |
| 11 | 254 | 2*127 |
| 12 | 411 | 3*137 |
| 13 | 665 | 5*7*19 |
| 14 | 1076 | 2*2*269 |
| 15 | 1741 | 1741 |
| 16 | 2817 | 3*3*313 |
| 17 | 4558 | 2*43*53 |
| 18 | 7375 | 5*5*5*59 |
| 19 | 11933 | 11933 |
| 20 | 19308 | 2*2*3*1609 |
| 21 | 31241 | 7*4463 |
| 22 | 50549 | 50549 |
| 23 | 81790 | 2*5*8179 |
| 24 | 132339 | 3*31*1423 |
| 25 | 214129 | 214129 |
| 26 | 346468 | 2*2*37*2341 |
| 27 | 560597 | 560597 |
| 28 | 907065 | 3*3*3*5*6719 |
| 29 | 1467662 | 2*7*79*1327 |
| 30 | 2374727 | 23*223*463 |
| 31 | 3842389 | 19*202231 |
| 32 | 6217116 | 2*2*3*379*1367 |
| 33 | 10059505 | 5*227*8863 |
| 34 | 16276621 | 16276621 |
| 35 | 26336126 | 2*641*20543 |
| 36 | 42612747 | 3*1637*8677 |
| 37 | 68948873 | 7*181*54419 |
| 38 | 111561620 | 2*2*5*5578081 |
| 39 | 180510493 | 180510493 |
| 40 | 292072113 | 3*3*32452457 |
| 41 | 472582606 | 2*1109*213067 |
| 42 | 764654719 | 67*2083*5479 |
| 43 | 1237237325 | 5*5*49489493 |
| 44 | 2001892044 | 2*2*3*53*3147629 |
| 45 | 3239129369 | 7*7*37*1786613 |

## THE NUMBER OF REPRESENTATIONS S(n) OF INTEGERS AS SUMS OF DISTINCT ELEMENTS OF THE ABBREVIATED FIBONACCI SEQUENCE

The abbreviated Fibonacci sequence lacking the first term is: 1,2,3,5,8,......This table can also be used to calculate the number of representations T(n) of integers as sums of distinct elements of the complete Fibonacci sequence: 1,1,2,3,5,8,.....by means of the relation:

$$T(n) = S(n) + S(n-1)$$

| n | S(n) | n | S(n) | n | S(n) | n | S(n) | n | S(n) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 45 | 6 | 89 | 5 | 133 | 6 | 177 | 2 |
| 2 | 1 | 46 | 2 | 90 | 5 | 134 | 9 | 178 | 8 |
| 3 | 2 | 47 | 5 | 91 | 4 | 135 | 3 | 179 | 8 |
| 4 | 1 | 48 | 5 | 92 | 8 | 136 | 7 | 180 | 6 |
| 5 | 2 | 49 | 3 | 93 | 4 | 137 | 7 | 181 | 12 |
| 6 | 2 | 50 | 6 | 94 | 7 | 138 | 4 | 182 | 6 |
| 7 | 1 | 51 | 3 | 95 | 7 | 139 | 8 | 183 | 10 |
| 8 | 3 | 52 | 4 | 96 | 3 | 140 | 4 | 184 | 10 |
| 9 | 2 | 53 | 4 | 97 | 9 | 141 | 5 | 185 | 4 |
| 10 | 2 | 54 | 1 | 98 | 6 | 142 | 5 | 186 | 12 |
| 11 | 3 | 55 | 5 | 99 | 6 | 143 | 1 | 187 | 8 |
| 12 | 1 | 56 | 4 | 100 | 9 | 144 | 6 | 188 | 8 |
| 13 | 3 | 57 | 4 | 101 | 3 | 145 | 5 | 189 | 12 |
| 14 | 3 | 58 | 7 | 102 | 8 | 146 | 5 | 190 | 4 |
| 15 | 2 | 59 | 3 | 103 | 8 | 147 | 9 | 191 | 10 |
| 16 | 4 | 60 | 6 | 104 | 5 | 148 | 4 | 192 | 10 |
| 17 | 2 | 61 | 6 | 105 | 10 | 149 | 8 | 193 | 6 |
| 18 | 3 | 62 | 3 | 106 | 5 | 150 | 8 | 194 | 12 |
| 19 | 3 | 63 | 8 | 107 | 7 | 151 | 4 | 195 | 6 |
| 20 | 1 | 64 | 5 | 108 | 7 | 152 | 11 | 196 | 8 |
| 21 | 4 | 65 | 5 | 109 | 2 | 153 | 7 | 197 | 8 |
| 22 | 3 | 66 | 7 | 110 | 8 | 154 | 7 | 198 | 2 |
| 23 | 3 | 67 | 2 | 111 | 6 | 155 | 10 | 199 | 9 |
| 24 | 5 | 68 | 6 | 112 | 6 | 156 | 3 | 200 | 7 |
| 25 | 2 | 69 | 6 | 113 | 10 | 157 | 9 | 201 | 7 |
| 26 | 4 | 70 | 4 | 114 | 4 | 158 | 9 | 202 | 12 |
| 27 | 4 | 71 | 8 | 115 | 8 | 159 | 6 | 203 | 5 |
| 28 | 2 | 72 | 4 | 116 | 8 | 160 | 12 | 204 | 10 |
| 29 | 5 | 73 | 6 | 117 | 4 | 161 | 6 | 205 | 10 |
| 30 | 3 | 74 | 6 | 118 | 10 | 162 | 9 | 206 | 5 |
| 31 | 3 | 75 | 2 | 119 | 6 | 163 | 9 | 207 | 13 |
| 32 | 4 | 76 | 7 | 120 | 6 | 164 | 3 | 208 | 8 |
| 33 | 1 | 77 | 5 | 121 | 8 | 165 | 11 | 209 | 8 |
| 34 | 4 | 78 | 5 | 122 | 2 | 166 | 8 | 210 | 11 |
| 35 | 4 | 79 | 8 | 123 | 7 | 167 | 8 | 211 | 3 |
| 36 | 3 | 80 | 3 | 124 | 7 | 168 | 13 | 212 | 9 |
| 37 | 6 | 81 | 6 | 125 | 5 | 169 | 5 | 213 | 9 |
| 38 | 3 | 82 | 6 | 126 | 10 | 170 | 10 | 214 | 6 |
| 39 | 5 | 83 | 3 | 127 | 5 | 171 | 10 | 215 | 12 |
| 40 | 5 | 84 | 7 | 128 | 8 | 172 | 5 | 216 | 6 |
| 41 | 2 | 85 | 4 | 129 | 8 | 173 | 12 | 217 | 9 |
| 42 | 6 | 86 | 4 | 130 | 3 | 174 | 7 | 218 | 9 |
| 43 | 4 | 87 | 5 | 131 | 9 | 175 | 7 | 219 | 3 |
| 44 | 4 | 88 | 1 | 132 | 6 | 176 | 9 | 220 | 10 |

THE NUMBER OF REPRESENTATIONS S(n) OF INTEGERS AS SUMS OF DISTINCT ELEMENTS OF THE ABBREVIATED FIBONACCI SEQUENCE

| n | S(n) | n | S(n) | n | S(n) | n | S(n) | n | S(n) |
|---|---|---|---|---|---|---|---|---|---|
| 221 | 7 | 271 | 8 | 321 | 2 | 371 | 5 | 421 | 12 |
| 222 | 7 | 272 | 13 | 322 | 9 | 372 | 10 | 422 | 18 |
| 223 | 11 | 273 | 13 | 323 | 9 | 373 | 5 | 423 | 6 |
| 224 | 4 | 274 | 5 | 324 | 7 | 374 | 6 | 424 | 15 |
| 225 | 8 | 275 | 15 | 325 | 14 | 375 | 6 | 425 | 15 |
| 226 | 8 | 276 | 10 | 326 | 7 | 376 | 1 | 426 | 9 |
| 227 | 4 | 277 | 10 | 327 | 12 | 377 | 7 | 427 | 18 |
| 228 | 9 | 278 | 15 | 328 | 12 | 378 | 6 | 428 | 9 |
| 229 | 5 | 279 | 5 | 329 | 5 | 379 | 6 | 429 | 12 |
| 230 | 5 | 280 | 12 | 330 | 15 | 380 | 11 | 430 | 12 |
| 231 | 6 | 281 | 12 | 331 | 10 | 381 | 5 | 431 | 3 |
| 232 | 1 | 282 | 7 | 332 | 10 | 382 | 10 | 432 | 14 |
| 233 | 6 | 283 | 14 | 333 | 15 | 383 | 10 | 433 | 11 |
| 234 | 6 | 284 | 7 | 334 | 5 | 384 | 5 | 434 | 11 |
| 235 | 5 | 285 | 9 | 335 | 13 | 385 | 14 | 435 | 19 |
| 236 | 10 | 286 | 9 | 336 | 13 | 386 | 9 | 436 | 8 |
| 237 | 5 | 287 | 2 | 337 | 8 | 387 | 9 | 437 | 16 |
| 238 | 9 | 288 | 10 | 338 | 16 | 388 | 13 | 438 | 16 |
| 239 | 9 | 289 | 8 | 339 | 8 | 389 | 4 | 439 | 8 |
| 240 | 4 | 290 | 8 | 340 | 11 | 390 | 12 | 440 | 21 |
| 241 | 12 | 291 | 14 | 341 | 11 | 391 | 12 | 441 | 13 |
| 242 | 8 | 292 | 6 | 342 | 3 | 392 | 8 | 442 | 13 |
| 243 | 8 | 293 | 12 | 343 | 12 | 393 | 16 | 443 | 18 |
| 244 | 12 | 294 | 12 | 344 | 9 | 394 | 8 | 444 | 5 |
| 245 | 4 | 295 | 6 | 345 | 9 | 395 | 12 | 445 | 15 |
| 246 | 11 | 296 | 16 | 346 | 15 | 396 | 12 | 446 | 15 |
| 247 | 11 | 297 | 10 | 347 | 6 | 397 | 4 | 447 | 10 |
| 248 | 7 | 298 | 10 | 348 | 12 | 398 | 15 | 448 | 20 |
| 249 | 14 | 299 | 14 | 349 | 12 | 399 | 11 | 449 | 10 |
| 250 | 7 | 300 | 4 | 350 | 6 | 400 | 11 | 450 | 15 |
| 251 | 10 | 301 | 12 | 351 | 15 | 401 | 18 | 451 | 15 |
| 252 | 10 | 302 | 12 | 352 | 9 | 402 | 7 | 452 | 5 |
| 253 | 3 | 303 | 8 | 353 | 9 | 403 | 14 | 453 | 17 |
| 254 | 12 | 304 | 16 | 354 | 12 | 404 | 14 | 454 | 12 |
| 255 | 9 | 305 | 8 | 355 | 3 | 405 | 7 | 455 | 12 |
| 256 | 9 | 306 | 12 | 356 | 10 | 406 | 17 | 456 | 19 |
| 257 | 15 | 307 | 12 | 357 | 10 | 407 | 10 | 457 | 7 |
| 258 | 6 | 308 | 4 | 358 | 7 | 408 | 10 | 458 | 14 |
| 259 | 12 | 309 | 14 | 359 | 14 | 409 | 13 | 459 | 14 |
| 260 | 12 | 310 | 10 | 360 | 7 | 410 | 3 | 460 | 7 |
| 261 | 6 | 311 | 10 | 361 | 11 | 411 | 12 | 461 | 16 |
| 262 | 15 | 312 | 16 | 362 | 11 | 412 | 12 | 462 | 9 |
| 263 | 9 | 313 | 6 | 363 | 4 | 413 | 9 | 463 | 9 |
| 264 | 9 | 314 | 12 | 364 | 12 | 414 | 18 | 464 | 11 |
| 265 | 12 | 315 | 12 | 365 | 8 | 415 | 9 | 465 | 2 |
| 266 | 3 | 316 | 6 | 366 | 8 | 416 | 15 | 466 | 10 |
| 267 | 11 | 317 | 14 | 367 | 12 | 417 | 15 | 467 | 10 |
| 268 | 11 | 318 | 8 | 368 | 4 | 418 | 6 | 468 | 8 |
| 269 | 8 | 319 | 8 | 369 | 9 | 419 | 18 | 469 | 16 |
| 270 | 16 | 320 | 10 | 370 | 9 | 420 | 12 | 470 | 8 |

THE NUMBER OF REPRESENTATIONS S(n) OF INTEGERS AS SUMS OF DISTINCT ELEMENTS OF THE ABBREVIATED FIBONACCI SEQUENCE

| n | S(n) | n | S(n) | n | S(n) | n | S(n) | n | S(n) |
|---|---|---|---|---|---|---|---|---|---|
| 471 | 14 | 521 | 11 | 571 | 18 | 621 | 15 | 671 | 18 |
| 472 | 14 | 522 | 9 | 572 | 9 | 622 | 5 | 672 | 9 |
| 473 | 6 | 523 | 9 | 573 | 12 | 623 | 14 | 673 | 24 |
| 474 | 18 | 524 | 16 | 574 | 12 | 624 | 14 | 674 | 15 |
| 475 | 12 | 525 | 7 | 575 | 3 | 625 | 9 | 675 | 15 |
| 476 | 12 | 526 | 14 | 576 | 13 | 626 | 18 | 676 | 21 |
| 477 | 18 | 527 | 14 | 577 | 10 | 627 | 9 | 677 | 6 |
| 478 | 6 | 528 | 7 | 578 | 10 | 628 | 13 | 678 | 18 |
| 479 | 16 | 529 | 19 | 579 | 17 | 629 | 13 | 679 | 18 |
| 480 | 16 | 530 | 12 | 580 | 7 | 630 | 4 | 680 | 12 |
| 481 | 10 | 531 | 12 | 581 | 14 | 631 | 16 | 681 | 24 |
| 482 | 20 | 532 | 17 | 582 | 14 | 632 | 12 | 682 | 12 |
| 483 | 10 | 533 | 5 | 583 | 7 | 633 | 12 | 683 | 18 |
| 484 | 14 | 534 | 15 | 584 | 18 | 634 | 20 | 684 | 18 |
| 485 | 14 | 535 | 15 | 585 | 11 | 635 | 8 | 685 | 6 |
| 486 | 4 | 536 | 10 | 586 | 11 | 636 | 16 | 686 | 21 |
| 487 | 16 | 537 | 20 | 587 | 15 | 637 | 16 | 687 | 15 |
| 488 | 12 | 538 | 10 | 588 | 4 | 638 | 8 | 688 | 15 |
| 489 | 12 | 539 | 15 | 589 | 12 | 639 | 20 | 689 | 24 |
| 490 | 20 | 540 | 15 | 590 | 12 | 640 | 12 | 690 | 9 |
| 491 | 8 | 541 | 5 | 591 | 8 | 641 | 12 | 691 | 18 |
| 492 | 16 | 542 | 18 | 592 | 16 | 642 | 16 | 692 | 18 |
| 493 | 16 | 543 | 13 | 593 | 8 | 643 | 4 | 693 | 9 |
| 494 | 8 | 544 | 13 | 594 | 12 | 644 | 15 | 694 | 21 |
| 495 | 20 | 545 | 21 | 595 | 12 | 645 | 15 | 695 | 12 |
| 496 | 12 | 546 | 8 | 596 | 4 | 646 | 11 | 696 | 12 |
| 497 | 12 | 547 | 16 | 597 | 13 | 647 | 22 | 697 | 15 |
| 498 | 16 | 548 | 16 | 598 | 9 | 648 | 11 | 698 | 3 |
| 499 | 4 | 549 | 8 | 599 | 9 | 649 | 18 | 699 | 14 |
| 500 | 14 | 550 | 19 | 600 | 14 | 650 | 18 | 700 | 14 |
| 501 | 14 | 551 | 11 | 601 | 5 | 651 | 7 | 701 | 11 |
| 502 | 10 | 552 | 11 | 602 | 10 | 652 | 21 | 702 | 22 |
| 503 | 20 | 553 | 14 | 603 | 10 | 653 | 14 | 703 | 11 |
| 504 | 10 | 554 | 3 | 604 | 5 | 654 | 14 | 704 | 19 |
| 505 | 16 | 555 | 12 | 605 | 11 | 655 | 21 | 705 | 19 |
| 506 | 16 | 556 | 12 | 606 | 6 | 656 | 7 | 706 | 8 |
| 507 | 6 | 557 | 9 | 607 | 6 | 657 | 17 | 707 | 24 |
| 508 | 18 | 558 | 18 | 608 | 7 | 658 | 17 | 708 | 16 |
| 509 | 12 | 559 | 9 | 609 | 1 | 659 | 10 | 709 | 16 |
| 510 | 12 | 560 | 15 | 610 | 7 | 660 | 20 | 710 | 24 |
| 511 | 18 | 561 | 15 | 611 | 7 | 661 | 10 | 711 | 8 |
| 512 | 6 | 562 | 6 | 612 | 6 | 662 | 13 | 712 | 21 |
| 513 | 14 | 563 | 18 | 613 | 12 | 663 | 13 | 713 | 21 |
| 514 | 14 | 564 | 12 | 614 | 6 | 664 | 3 | 714 | 13 |
| 515 | 8 | 565 | 12 | 615 | 11 | 665 | 15 | 715 | 26 |
| 516 | 16 | 566 | 18 | 616 | 11 | 666 | 12 | 716 | 13 |
| 517 | 8 | 567 | 6 | 617 | 5 | 667 | 12 | 717 | 18 |
| 518 | 10 | 568 | 15 | 618 | 15 | 668 | 21 | 718 | 18 |
| 519 | 10 | 569 | 15 | 619 | 10 | 669 | 9 | 719 | 5 |
| 520 | 2 | 570 | 9 | 620 | 10 | 670 | 18 | 720 | 20 |

THE NUMBER OF REPRESENTATIONS S(n) OF INTEGERS AS SUMS OF DISTINCT ELEMENTS OF THE ABBREVIATED FIBONACCI SEQUENCE

| n | S(n) | n | S(n) | n | S(n) | n | S(n) | n | S(n) |
|---|---|---|---|---|---|---|---|---|---|
| 721 | 15 | 771 | 12 | 821 | 6 | 871 | 10 | 921 | 15 |
| 722 | 15 | 772 | 18 | 822 | 18 | 872 | 25 | 922 | 24 |
| 723 | 25 | 773 | 18 | 823 | 18 | 873 | 15 | 923 | 9 |
| 724 | 10 | 774 | 6 | 824 | 12 | 874 | 15 | 924 | 18 |
| 725 | 20 | 775 | 22 | 825 | 24 | 875 | 20 | 925 | 18 |
| 726 | 20 | 776 | 16 | 826 | 12 | 876 | 5 | 926 | 9 |
| 727 | 10 | 777 | 16 | 827 | 18 | 877 | 18 | 927 | 21 |
| 728 | 25 | 778 | 26 | 828 | 18 | 878 | 18 | 928 | 12 |
| 729 | 15 | 779 | 10 | 829 | 6 | 879 | 13 | 929 | 12 |
| 730 | 15 | 780 | 20 | 830 | 20 | 880 | 26 | 930 | 15 |
| 731 | 20 | 781 | 20 | 831 | 14 | 881 | 13 | 931 | 3 |
| 732 | 5 | 782 | 10 | 832 | 14 | 882 | 21 | 932 | 13 |
| 733 | 17 | 783 | 24 | 833 | 22 | 883 | 21 | 933 | 13 |
| 734 | 17 | 784 | 14 | 834 | 8 | 884 | 8 | 934 | 10 |
| 735 | 12 | 785 | 14 | 835 | 16 | 885 | 24 | 935 | 20 |
| 736 | 24 | 786 | 18 | 836 | 16 | 886 | 16 | 936 | 10 |
| 737 | 12 | 787 | 4 | 837 | 8 | 887 | 16 | 937 | 17 |
| 738 | 19 | 788 | 16 | 838 | 18 | 888 | 24 | 938 | 17 |
| 739 | 19 | 789 | 16 | 839 | 10 | 889 | 8 | 939 | 7 |
| 740 | 7 | 790 | 12 | 840 | 10 | 890 | 19 | 940 | 21 |
| 741 | 21 | 791 | 24 | 841 | 12 | 891 | 19 | 941 | 14 |
| 742 | 14 | 792 | 12 | 842 | 2 | 892 | 11 | 942 | 14 |
| 743 | 14 | 793 | 20 | 843 | 11 | 893 | 22 | 943 | 21 |
| 744 | 21 | 794 | 20 | 844 | 11 | 894 | 11 | 944 | 7 |
| 745 | 7 | 795 | 8 | 845 | 9 | 895 | 14 | 945 | 18 |
| 746 | 16 | 796 | 24 | 846 | 18 | 896 | 14 | 946 | 18 |
| 747 | 16 | 797 | 16 | 847 | 9 | 897 | 3 | 947 | 11 |
| 748 | 9 | 798 | 16 | 848 | 16 | 898 | 15 | 948 | 22 |
| 749 | 18 | 799 | 24 | 849 | 16 | 899 | 12 | 949 | 11 |
| 750 | 9 | 800 | 8 | 850 | 7 | 900 | 12 | 950 | 15 |
| 751 | 11 | 801 | 20 | 851 | 21 | 901 | 21 | 951 | 15 |
| 752 | 11 | 802 | 20 | 852 | 14 | 902 | 9 | 952 | 4 |
| 753 | 2 | 803 | 12 | 853 | 14 | 903 | 18 | 953 | 16 |
| 754 | 12 | 804 | 24 | 854 | 21 | 904 | 18 | 954 | 12 |
| 755 | 10 | 805 | 12 | 855 | 7 | 905 | 9 | 955 | 12 |
| 756 | 10 | 806 | 16 | 856 | 19 | 906 | 24 | 956 | 20 |
| 757 | 18 | 807 | 16 | 587 | 19 | 907 | 15 | 957 | 8 |
| 758 | 8 | 808 | 4 | 858 | 12 | 908 | 15 | 958 | 16 |
| 759 | 16 | 809 | 18 | 859 | 24 | 909 | 21 | 959 | 16 |
| 760 | 16 | 810 | 14 | 860 | 12 | 910 | 6 | 960 | 8 |
| 761 | 8 | 811 | 14 | 861 | 17 | 911 | 18 | 961 | 20 |
| 762 | 22 | 812 | 24 | 862 | 17 | 912 | 18 | 962 | 12 |
| 763 | 14 | 813 | 10 | 863 | 5 | 913 | 12 | 963 | 12 |
| 764 | 14 | 814 | 20 | 864 | 20 | 914 | 24 | 964 | 16 |
| 765 | 20 | 815 | 20 | 865 | 15 | 915 | 12 | 965 | 4 |
| 766 | 6 | 816 | 10 | 866 | 15 | 916 | 18 | 966 | 13 |
| 767 | 18 | 817 | 26 | 867 | 25 | 917 | 18 | 967 | 13 |
| 768 | 18 | 818 | 16 | 868 | 10 | 918 | 6 | 968 | 9 |
| 769 | 12 | 819 | 16 | 869 | 20 | 919 | 21 | 969 | 18 |
| 770 | 24 | 820 | 22 | 870 | 20 | 920 | 15 | 970 | 9 |

## THE NUMBER OF REPRESENTATIONS Q(n) OF INTEGERS AS SUMS OF THE ELEMENTS OF THE LUCAS SEQUENCE

The Lucas sequence beginning with the first term is : 1,3,4,7,11.... The enlarged Lucas sequence including the zero term is: 2,1,3,4,7,11,.... If the number of representations by distinct elements of the enlarged Lucas sequence is designated R(n), this quantity can be found using the relation:

$$R(n) = Q(n) + Q(n-2)$$

| n | Q(n) | n | Q(n) | n | Q(n) | n | Q(n) | n | Q(n) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 41 | 3 | 81 | 3 | 121 | 1 | 161 | 0 |
| 2 | 0 | 42 | 0 | 82 | 0 | 122 | 0 | 162 | 4 |
| 3 | 1 | 43 | 3 | 83 | 6 | 123 | 5 | 163 | 10 |
| 4 | 2 | 44 | 4 | 84 | 6 | 124 | 5 | 164 | 6 |
| 5 | 1 | 45 | 1 | 85 | 0 | 125 | 0 | 165 | 0 |
| 6 | 0 | 46 | 0 | 86 | 3 | 126 | 4 | 166 | 6 |
| 7 | 2 | 47 | 4 | 87 | 8 | 127 | 8 | 167 | 8 |
| 8 | 2 | 48 | 4 | 88 | 5 | 128 | 4 | 168 | 2 |
| 9 | 0 | 49 | 0 | 89 | 0 | 129 | 0 | 169 | 0 |
| 10 | 1 | 50 | 3 | 90 | 5 | 130 | 7 | 170 | 7 |
| 11 | 3 | 51 | 6 | 91 | 7 | 131 | 7 | 171 | 7 |
| 12 | 2 | 52 | 3 | 92 | 2 | 132 | 0 | 172 | 0 |
| 13 | 0 | 53 | 0 | 93 | 0 | 133 | 3 | 173 | 5 |
| 14 | 2 | 54 | 5 | 94 | 6 | 134 | 9 | 174 | 10 |
| 15 | 3 | 55 | 5 | 95 | 6 | 135 | 6 | 175 | 5 |
| 16 | 1 | 56 | 0 | 96 | 0 | 136 | 0 | 176 | 0 |
| 17 | 0 | 57 | 2 | 97 | 4 | 137 | 6 | 177 | 8 |
| 18 | 3 | 58 | 6 | 98 | 8 | 138 | 9 | 178 | 8 |
| 19 | 3 | 59 | 4 | 99 | 4 | 139 | 3 | 179 | 0 |
| 20 | 0 | 60 | 0 | 100 | 0 | 140 | 0 | 180 | 3 |
| 21 | 2 | 61 | 4 | 101 | 6 | 141 | 8 | 181 | 9 |
| 22 | 4 | 62 | 6 | 102 | 6 | 142 | 8 | 182 | 6 |
| 23 | 2 | 63 | 2 | 103 | 0 | 143 | 0 | 183 | 0 |
| 24 | 0 | 64 | 0 | 104 | 2 | 144 | 5 | 184 | 6 |
| 25 | 3 | 65 | 5 | 105 | 7 | 145 | 10 | 185 | 9 |
| 26 | 3 | 66 | 5 | 106 | 5 | 146 | 5 | 186 | 3 |
| 27 | 0 | 67 | 0 | 107 | 0 | 147 | 0 | 187 | 0 |
| 28 | 1 | 68 | 3 | 108 | 5 | 148 | 7 | 188 | 7 |
| 29 | 4 | 69 | 6 | 109 | 8 | 149 | 7 | 189 | 7 |
| 30 | 3 | 70 | 3 | 110 | 3 | 150 | 0 | 190 | 0 |
| 31 | 0 | 71 | 0 | 111 | 0 | 151 | 2 | 191 | 4 |
| 32 | 3 | 72 | 4 | 112 | 6 | 152 | 8 | 192 | 8 |
| 33 | 5 | 73 | 4 | 113 | 6 | 153 | 6 | 193 | 4 |
| 34 | 2 | 74 | 0 | 114 | 0 | 154 | 0 | 194 | 0 |
| 35 | 0 | 75 | 1 | 115 | 3 | 155 | 6 | 195 | 5 |
| 36 | 4 | 76 | 5 | 116 | 7 | 156 | 10 | 196 | 5 |
| 37 | 4 | 77 | 4 | 117 | 4 | 157 | 4 | 197 | 0 |
| 38 | 0 | 78 | 0 | 118 | 0 | 158 | 0 | 198 | 1 |
| 39 | 2 | 79 | 4 | 119 | 4 | 159 | 8 | 199 | 6 |
| 40 | 5 | 80 | 7 | 120 | 5 | 160 | 8 | 200 | 5 |

THE NUMBER OF REPRESENTATIONS Q(n) OF INTEGERS AS SUMS OF THE ELEMENTS OF THE LUCAS SEQUENCE

| n | Q(n) | n | Q(n) | n | Q(n) | n | Q(n) | n | Q(n) |
|---|---|---|---|---|---|---|---|---|---|
| 201 | 0 | 251 | 6 | 301 | 9 | 351 | 12 | 401 | 8 |
| 202 | 5 | 252 | 0 | 302 | 0 | 352 | 9 | 402 | 14 |
| 203 | 9 | 253 | 10 | 303 | 3 | 353 | 0 | 403 | 6 |
| 204 | 4 | 254 | 10 | 304 | 10 | 354 | 9 | 404 | 0 |
| 205 | 0 | 255 | 0 | 305 | 7 | 355 | 15 | 405 | 12 |
| 206 | 8 | 256 | 4 | 306 | 0 | 356 | 6 | 406 | 12 |
| 207 | 8 | 257 | 12 | 307 | 7 | 357 | 0 | 407 | 0 |
| 208 | 0 | 258 | 8 | 308 | 11 | 358 | 12 | 408 | 6 |
| 209 | 4 | 259 | 0 | 309 | 4 | 359 | 12 | 409 | 16 |
| 210 | 11 | 260 | 8 | 310 | 0 | 360 | 0 | 410 | 10 |
| 211 | 7 | 261 | 12 | 311 | 8 | 361 | 6 | 411 | 0 |
| 212 | 0 | 262 | 4 | 312 | 8 | 362 | 15 | 412 | 10 |
| 213 | 7 | 263 | 0 | 313 | 0 | 363 | 9 | 413 | 14 |
| 214 | 10 | 264 | 10 | 314 | 4 | 364 | 0 | 414 | 4 |
| 215 | 3 | 265 | 10 | 315 | 9 | 365 | 9 | 415 | 0 |
| 216 | 0 | 266 | 0 | 316 | 5 | 366 | 12 | 416 | 12 |
| 217 | 9 | 267 | 6 | 317 | 0 | 367 | 3 | 417 | 12 |
| 218 | 9 | 268 | 12 | 318 | 5 | 368 | 0 | 418 | 0 |
| 219 | 0 | 269 | 6 | 319 | 6 | 369 | 11 | 419 | 8 |
| 220 | 6 | 270 | 0 | 320 | 1 | 370 | 11 | 420 | 16 |
| 221 | 12 | 271 | 8 | 321 | 0 | 371 | 0 | 421 | 8 |
| 222 | 6 | 272 | 8 | 322 | 6 | 372 | 8 | 422 | 0 |
| 223 | 0 | 273 | 0 | 323 | 6 | 373 | 16 | 423 | 12 |
| 224 | 9 | 274 | 2 | 324 | 0 | 374 | 8 | 424 | 12 |
| 225 | 9 | 275 | 9 | 325 | 5 | 375 | 0 | 425 | 0 |
| 226 | 0 | 276 | 7 | 326 | 10 | 376 | 13 | 426 | 4 |
| 227 | 3 | 277 | 0 | 327 | 5 | 377 | 13 | 427 | 14 |
| 228 | 11 | 278 | 7 | 328 | 0 | 378 | 0 | 428 | 10 |
| 229 | 8 | 279 | 12 | 329 | 9 | 379 | 5 | 429 | 0 |
| 230 | 0 | 280 | 5 | 330 | 9 | 380 | 15 | 430 | 10 |
| 231 | 8 | 281 | 0 | 331 | 0 | 381 | 10 | 431 | 16 |
| 232 | 13 | 282 | 10 | 332 | 4 | 382 | 0 | 432 | 6 |
| 233 | 5 | 283 | 10 | 333 | 12 | 383 | 10 | 433 | 0 |
| 234 | 0 | 284 | 0 | 334 | 8 | 384 | 15 | 434 | 12 |
| 235 | 10 | 285 | 5 | 335 | 0 | 385 | 5 | 435 | 12 |
| 236 | 10 | 286 | 13 | 336 | 8 | 386 | 0 | 436 | 0 |
| 237 | 0 | 287 | 8 | 337 | 12 | 387 | 12 | 437 | 6 |
| 238 | 5 | 288 | 0 | 338 | 4 | 388 | 12 | 438 | 14 |
| 239 | 12 | 289 | 8 | 339 | 0 | 389 | 0 | 439 | 8 |
| 240 | 7 | 290 | 11 | 340 | 11 | 390 | 7 | 440 | 0 |
| 241 | 0 | 291 | 3 | 341 | 11 | 391 | 14 | 441 | 8 |
| 242 | 7 | 292 | 0 | 342 | 0 | 392 | 7 | 442 | 10 |
| 243 | 9 | 293 | 9 | 343 | 7 | 393 | 0 | 443 | 2 |
| 244 | 2 | 294 | 9 | 344 | 14 | 394 | 9 | 444 | 0 |
| 245 | 0 | 295 | 0 | 345 | 7 | 395 | 9 | 445 | 9 |
| 246 | 8 | 296 | 6 | 346 | 0 | 396 | 0 | 446 | 9 |
| 247 | 8 | 297 | 12 | 347 | 10 | 397 | 2 | 447 | 0 |
| 248 | 0 | 298 | 6 | 348 | 10 | 398 | 10 | 448 | 7 |
| 249 | 6 | 299 | 0 | 349 | 0 | 399 | 8 | 449 | 14 |
| 250 | 12 | 300 | 9 | 350 | 3 | 400 | 0 | 450 | 7 |

THE NUMBER OF REPRESENTATIONS Q(n) OF INTEGERS AS SUMS OF THE ELEMENTS OF THE LUCAS SEQUENCE

| n | Q(n) | n | Q(n) | n | Q(n) | n | Q(n) | n | Q(n) |
|---|---|---|---|---|---|---|---|---|---|
| 451 | 0 | 501 | 0 | 551 | 11 | 601 | 19 | 651 | 14 |
| 452 | 12 | 502 | 4 | 552 | 0 | 602 | 8 | 652 | 14 |
| 453 | 12 | 503 | 12 | 553 | 11 | 603 | 0 | 653 | 0 |
| 454 | 0 | 504 | 8 | 554 | 18 | 604 | 16 | 654 | 6 |
| 455 | 5 | 505 | 0 | 555 | 7 | 605 | 16 | 655 | 18 |
| 456 | 15 | 506 | 8 | 556 | 0 | 606 | 0 | 656 | 12 |
| 457 | 10 | 507 | 12 | 557 | 14 | 607 | 8 | 657 | 0 |
| 458 | 0 | 508 | 4 | 558 | 14 | 608 | 21 | 658 | 12 |
| 459 | 10 | 509 | 0 | 559 | 0 | 609 | 13 | 659 | 18 |
| 460 | 15 | 510 | 9 | 560 | 7 | 610 | 0 | 660 | 6 |
| 461 | 5 | 511 | 9 | 561 | 17 | 611 | 13 | 661 | 0 |
| 462 | 0 | 512 | 0 | 562 | 10 | 612 | 18 | 662 | 16 |
| 463 | 13 | 513 | 5 | 563 | 0 | 613 | 5 | 663 | 16 |
| 464 | 13 | 514 | 10 | 564 | 10 | 614 | 0 | 664 | 0 |
| 465 | 0 | 515 | 5 | 565 | 13 | 615 | 15 | 665 | 10 |
| 466 | 8 | 516 | 0 | 566 | 3 | 616 | 15 | 666 | 20 |
| 467 | 16 | 517 | 6 | 567 | 0 | 617 | 0 | 667 | 10 |
| 468 | 8 | 518 | 6 | 568 | 12 | 618 | 10 | 668 | 0 |
| 469 | 0 | 519 | 0 | 569 | 12 | 619 | 20 | 669 | 14 |
| 470 | 11 | 520 | 1 | 570 | 0 | 620 | 10 | 670 | 14 |
| 471 | 11 | 521 | 7 | 571 | 9 | 621 | 0 | 671 | 0 |
| 472 | 0 | 522 | 6 | 572 | 18 | 622 | 15 | 672 | 4 |
| 473 | 3 | 523 | 0 | 573 | 9 | 623 | 15 | 673 | 16 |
| 474 | 12 | 524 | 6 | 574 | 0 | 624 | 0 | 674 | 12 |
| 475 | 9 | 525 | 11 | 575 | 15 | 625 | 5 | 675 | 0 |
| 476 | 0 | 526 | 5 | 576 | 15 | 626 | 17 | 676 | 12 |
| 477 | 9 | 527 | 0 | 577 | 0 | 627 | 12 | 677 | 20 |
| 478 | 15 | 528 | 10 | 578 | 6 | 628 | 0 | 678 | 8 |
| 479 | 6 | 529 | 10 | 579 | 18 | 629 | 12 | 679 | 0 |
| 480 | 0 | 530 | 0 | 580 | 12 | 630 | 19 | 680 | 16 |
| 481 | 12 | 531 | 5 | 581 | 0 | 631 | 7 | 681 | 16 |
| 482 | 12 | 532 | 14 | 582 | 12 | 632 | 0 | 682 | 0 |
| 483 | 0 | 533 | 9 | 583 | 18 | 633 | 14 | 683 | 8 |
| 484 | 6 | 534 | 0 | 584 | 6 | 634 | 14 | 684 | 20 |
| 485 | 15 | 535 | 9 | 585 | 0 | 635 | 0 | 685 | 12 |
| 486 | 9 | 536 | 13 | 586 | 15 | 636 | 7 | 686 | 0 |
| 487 | 0 | 537 | 4 | 587 | 15 | 637 | 16 | 687 | 12 |
| 488 | 9 | 538 | 0 | 588 | 0 | 638 | 9 | 688 | 16 |
| 489 | 12 | 539 | 12 | 589 | 9 | 639 | 0 | 689 | 4 |
| 490 | 3 | 540 | 12 | 590 | 18 | 640 | 9 | 690 | 0 |
| 491 | 0 | 541 | 0 | 591 | 9 | 641 | 11 | 691 | 14 |
| 492 | 10 | 542 | 8 | 592 | 0 | 642 | 2 | 692 | 14 |
| 493 | 10 | 543 | 16 | 593 | 12 | 643 | 0 | 693 | 0 |
| 494 | 0 | 544 | 8 | 594 | 12 | 644 | 10 | 694 | 10 |
| 495 | 7 | 545 | 0 | 595 | 0 | 645 | 10 | 695 | 20 |
| 496 | 14 | 546 | 12 | 596 | 3 | 646 | 0 | 696 | 10 |
| 497 | 7 | 547 | 12 | 597 | 14 | 647 | 8 | 697 | 0 |
| 498 | 0 | 548 | 0 | 598 | 11 | 648 | 16 | 698 | 16 |
| 499 | 11 | 549 | 4 | 599 | 0 | 649 | 8 | 699 | 16 |
| 500 | 11 | 550 | 15 | 600 | 11 | 650 | 0 | 700 | 0 |

THE NUMBER OF REPRESENTATIONS Q(n) OF INTEGERS AS SUMS OF

THE ELEMENTS OF THE LUCAS SEQUENCE

| n | Q(n) | n | Q(n) | n | Q(n) | n | Q(n) | n | Q(n) |
|---|---|---|---|---|---|---|---|---|---|
| 701 | 6 | 751 | 0 | 801 | 7 | 851 | 11 | 901 | 21 |
| 702 | 18 | 752 | 13 | 802 | 0 | 952 | 0 | 902 | 14 |
| 703 | 12 | 753 | 21 | 803 | 14 | 853 | 5 | 903 | 0 |
| 704 | 0 | 754 | 8 | 804 | 14 | 854 | 15 | 904 | 14 |
| 705 | 12 | 755 | 0 | 805 | 0 | 855 | 10 | 905 | 21 |
| 706 | 18 | 756 | 16 | 806 | 7 | 856 | 0 | 906 | 7 |
| 707 | 6 | 757 | 16 | 807 | 18 | 857 | 10 | 907 | 0 |
| 708 | 0 | 758 | 0 | 808 | 11 | 858 | 15 | 908 | 17 |
| 709 | 14 | 759 | 8 | 809 | 0 | 859 | 5 | 909 | 17 |
| 710 | 14 | 760 | 19 | 810 | 11 | 860 | 0 | 910 | 0 |
| 711 | 0 | 761 | 11 | 811 | 15 | 861 | 14 | 911 | 10 |
| 712 | 8 | 762 | 0 | 812 | 4 | 862 | 14 | 912 | 20 |
| 713 | 16 | 763 | 11 | 813 | 0 | 863 | 0 | 913 | 10 |
| 714 | 8 | 764 | 14 | 814 | 12 | 864 | 9 | 914 | 0 |
| 715 | 0 | 765 | 3 | 815 | 12 | 865 | 18 | 915 | 13 |
| 716 | 10 | 766 | 0 | 816 | 0 | 866 | 9 | 916 | 13 |
| 717 | 10 | 767 | 12 | 817 | 8 | 867 | 0 | 917 | 0 |
| 718 | 0 | 768 | 12 | 818 | 16 | 868 | 13 | 918 | 3 |
| 719 | 2 | 769 | 0 | 819 | 8 | 869 | 13 | 919 | 15 |
| 720 | 11 | 770 | 9 | 820 | 0 | 870 | 0 | 920 | 12 |
| 721 | 9 | 771 | 18 | 821 | 12 | 871 | 4 | 921 | 0 |
| 722 | 0 | 772 | 9 | 822 | 12 | 872 | 16 | 922 | 12 |
| 723 | 9 | 773 | 0 | 823 | 0 | 873 | 12 | 923 | 21 |
| 724 | 16 | 774 | 15 | 824 | 4 | 874 | 0 | 924 | 9 |
| 725 | 7 | 775 | 15 | 825 | 13 | 875 | 12 | 925 | 0 |
| 726 | 0 | 776 | 0 | 826 | 9 | 876 | 20 | 926 | 18 |
| 727 | 14 | 777 | 6 | 827 | 0 | 877 | 8 | 927 | 18 |
| 728 | 14 | 778 | 18 | 828 | 9 | 878 | 0 | 928 | 0 |
| 729 | 0 | 779 | 12 | 829 | 14 | 879 | 16 | 929 | 9 |
| 730 | 7 | 780 | 0 | 830 | 5 | 880 | 16 | 930 | 24 |
| 731 | 19 | 781 | 12 | 831 | 0 | 881 | 0 | 931 | 15 |
| 732 | 12 | 782 | 18 | 832 | 10 | 882 | 8 | 932 | 0 |
| 733 | 0 | 783 | 6 | 833 | 10 | 883 | 20 | 933 | 15 |
| 734 | 12 | 784 | 0 | 834 | 0 | 884 | 12 | 934 | 21 |
| 735 | 17 | 785 | 15 | 835 | 5 | 885 | 0 | 935 | 6 |
| 736 | 5 | 786 | 15 | 836 | 11 | 886 | 12 | 936 | 0 |
| 737 | 0 | 787 | 0 | 837 | 6 | 887 | 16 | 937 | 18 |
| 738 | 15 | 788 | 9 | 838 | 0 | 888 | 4 | 938 | 18 |
| 739 | 15 | 789 | 18 | 839 | 6 | 889 | 0 | 939 | 0 |
| 740 | 0 | 790 | 9 | 840 | 7 | 890 | 15 | 940 | 12 |
| 741 | 10 | 791 | 0 | 841 | 1 | 891 | 15 | 941 | 24 |
| 742 | 20 | 792 | 12 | 842 | 0 | 892 | 0 | 942 | 12 |
| 743 | 10 | 793 | 12 | 843 | 7 | 893 | 11 | 943 | 0 |
| 744 | 0 | 794 | 0 | 844 | 7 | 894 | 22 | 944 | 18 |
| 745 | 15 | 795 | 3 | 845 | 0 | 895 | 11 | 945 | 18 |
| 746 | 15 | 796 | 13 | 846 | 6 | 896 | 0 | 946 | 0 |
| 747 | 0 | 797 | 10 | 847 | 12 | 897 | 18 | 947 | 6 |
| 748 | 5 | 798 | 0 | 848 | 6 | 898 | 18 | 948 | 21 |
| 749 | 18 | 799 | 10 | 849 | 0 | 899 | 0 | 949 | 15 |
| 750 | 13 | 800 | 17 | 850 | 11 | 900 | 7 | 950 | 0 |

LIST OF INTEGERS NOT REPRESENTABLE BY

THE TRUNCATED FIBONACCI SEQUENCE 2,3,5,8,...

| N | C(N) | N | C(N) | N | C(N) |
|---|---|---|---|---|---|
| 1 | 1 | 41 | 106 | 81 | 211 |
| 2 | 4 | 42 | 108 | 82 | 213 |
| 3 | 6 | 43 | 111 | 83 | 216 |
| 4 | 9 | 44 | 114 | 84 | 218 |
| 5 | 12 | 45 | 116 | 85 | 221 |
| 6 | 14 | 46 | 119 | 86 | 224 |
| 7 | 17 | 47 | 122 | 87 | 226 |
| 8 | 19 | 48 | 124 | 88 | 229 |
| 9 | 22 | 49 | 127 | 89 | 232 |
| 10 | 25 | 50 | 129 | 90 | 234 |
| 11 | 27 | 51 | 132 | 91 | 237 |
| 12 | 30 | 52 | 135 | 92 | 239 |
| 13 | 33 | 53 | 137 | 93 | 242 |
| 14 | 35 | 54 | 140 | 94 | 245 |
| 15 | 38 | 55 | 142 | 95 | 247 |
| 16 | 40 | 56 | 145 | 96 | 250 |
| 17 | 43 | 57 | 148 | 97 | 252 |
| 18 | 46 | 58 | 150 | 98 | 255 |
| 19 | 48 | 59 | 153 | 99 | 258 |
| 20 | 51 | 60 | 156 | 100 | 260 |
| 21 | 53 | 61 | 158 | 101 | 263 |
| 22 | 56 | 62 | 161 | 102 | 266 |
| 23 | 59 | 63 | 163 | 103 | 268 |
| 24 | 61 | 64 | 166 | 104 | 271 |
| 25 | 64 | 65 | 169 | 105 | 273 |
| 26 | 67 | 66 | 171 | 106 | 276 |
| 27 | 69 | 67 | 174 | 107 | 279 |
| 28 | 72 | 68 | 177 | 108 | 281 |
| 29 | 74 | 69 | 179 | 109 | 284 |
| 30 | 77 | 70 | 182 | 110 | 286 |
| 31 | 80 | 71 | 184 | 111 | 289 |
| 32 | 82 | 72 | 187 | 112 | 292 |
| 33 | 85 | 73 | 190 | 113 | 294 |
| 34 | 88 | 74 | 192 | 114 | 297 |
| 35 | 90 | 75 | 195 | 115 | 300 |
| 36 | 93 | 76 | 197 | 116 | 302 |
| 37 | 95 | 77 | 200 | 117 | 305 |
| 38 | 98 | 78 | 203 | 118 | 307 |
| 39 | 101 | 79 | 205 | 119 | 310 |
| 40 | 103 | 80 | 208 | 120 | 313 |

LIST OF INTEGERS NOT REPRESENTABLE BY

THE TRUNCATED FIBONACCI SEQUENCE 2,3,5,8,...

| N | C(N) | N | C(N) | N | C(N) |
|---|---|---|---|---|---|
| 121 | 315 | 161 | 420 | 201 | 525 |
| 122 | 318 | 162 | 423 | 202 | 527 |
| 123 | 321 | 163 | 425 | 203 | 530 |
| 124 | 323 | 164 | 428 | 204 | 533 |
| 125 | 326 | 165 | 430 | 205 | 535 |
| 126 | 328 | 166 | 433 | 206 | 538 |
| 127 | 331 | 167 | 436 | 207 | 540 |
| 128 | 334 | 168 | 438 | 208 | 543 |
| 129 | 336 | 169 | 441 | 209 | 546 |
| 130 | 339 | 170 | 444 | 210 | 548 |
| 131 | 341 | 171 | 446 | 211 | 551 |
| 132 | 344 | 172 | 449 | 212 | 554 |
| 133 | 347 | 173 | 451 | 213 | 556 |
| 134 | 349 | 174 | 454 | 214 | 559 |
| 135 | 352 | 175 | 457 | 215 | 561 |
| 136 | 355 | 176 | 459 | 216 | 564 |
| 137 | 357 | 177 | 462 | 217 | 567 |
| 138 | 360 | 178 | 465 | 218 | 569 |
| 139 | 362 | 179 | 467 | 219 | 572 |
| 140 | 365 | 180 | 470 | 220 | 574 |
| 141 | 368 | 181 | 472 | 221 | 577 |
| 142 | 370 | 182 | 475 | 222 | 580 |
| 143 | 373 | 183 | 478 | 223 | 582 |
| 144 | 375 | 184 | 480 | 224 | 585 |
| 145 | 378 | 185 | 483 | 225 | 588 |
| 146 | 381 | 186 | 485 | 226 | 590 |
| 147 | 383 | 187 | 488 | 227 | 593 |
| 148 | 386 | 188 | 491 | 228 | 595 |
| 149 | 389 | 189 | 493 | 229 | 598 |
| 150 | 391 | 190 | 496 | 230 | 601 |
| 151 | 394 | 191 | 499 | 231 | 603 |
| 152 | 396 | 192 | 501 | 232 | 606 |
| 153 | 399 | 193 | 504 | 233 | 609 |
| 154 | 402 | 194 | 506 | 234 | 611 |
| 155 | 404 | 195 | 509 | 235 | 614 |
| 156 | 407 | 196 | 512 | 236 | 616 |
| 157 | 410 | 197 | 514 | 237 | 619 |
| 158 | 412 | 198 | 517 | 238 | 622 |
| 159 | 415 | 199 | 519 | 239 | 624 |
| 160 | 417 | 200 | 522 | 240 | 627 |

LIST OF INTEGERS NOT REPRESENTABLE BY

THE TRUNCATED FIBONACCI SEQUENCE 2,3,5,8,...

| N | C(N) | N | C(N) | N | C(N) |
|---|---|---|---|---|---|
| 241 | 629 | 281 | 734 | 321 | 839 |
| 242 | 632 | 282 | 737 | 322 | 842 |
| 243 | 635 | 283 | 739 | 323 | 844 |
| 244 | 637 | 284 | 742 | 324 | 847 |
| 245 | 640 | 285 | 745 | 325 | 849 |
| 246 | 643 | 286 | 747 | 326 | 852 |
| 247 | 645 | 287 | 750 | 327 | 855 |
| 248 | 648 | 288 | 752 | 328 | 857 |
| 249 | 650 | 289 | 755 | 329 | 860 |
| 250 | 653 | 290 | 758 | 330 | 862 |
| 251 | 656 | 291 | 760 | 331 | 865 |
| 252 | 658 | 292 | 763 | 332 | 868 |
| 253 | 661 | 293 | 766 | 333 | 870 |
| 254 | 663 | 294 | 768 | 334 | 873 |
| 255 | 666 | 295 | 771 | 335 | 876 |
| 256 | 669 | 296 | 773 | 336 | 878 |
| 257 | 671 | 297 | 776 | 337 | 881 |
| 258 | 674 | 298 | 779 | 338 | 883 |
| 259 | 677 | 299 | 781 | 339 | 886 |
| 260 | 679 | 300 | 784 | 340 | 889 |
| 261 | 682 | 301 | 787 | 341 | 891 |
| 262 | 684 | 302 | 789 | 342 | 894 |
| 263 | 687 | 303 | 792 | 343 | 896 |
| 264 | 690 | 304 | 794 | 344 | 899 |
| 265 | 692 | 305 | 797 | 345 | 902 |
| 266 | 695 | 306 | 800 | 346 | 904 |
| 267 | 698 | 307 | 802 | 347 | 907 |
| 268 | 700 | 308 | 805 | 348 | 910 |
| 269 | 703 | 309 | 807 | 349 | 912 |
| 270 | 705 | 310 | 810 | 350 | 915 |
| 271 | 708 | 311 | 813 | 351 | 917 |
| 272 | 711 | 312 | 815 | 352 | 920 |
| 273 | 713 | 313 | 818 | 353 | 923 |
| 274 | 716 | 314 | 821 | 354 | 925 |
| 275 | 718 | 315 | 823 | 355 | 928 |
| 276 | 721 | 316 | 826 | 356 | 931 |
| 277 | 724 | 317 | 828 | 357 | 933 |
| 278 | 726 | 318 | 831 | 358 | 936 |
| 279 | 729 | 319 | 834 | 359 | 938 |
| 280 | 732 | 320 | 836 | 360 | 941 |

## INTEGERS NOT REPRESENTABLE BY THE TRUNCATED TRIBONACCI SEQUENCE

The following table gives the list of integers in sequence not representable by the truncated Tribonacci sequence $T_2 = 2$, $T_3 = 4$, $T_4 = 7$, with additional terms generated by

$$T_{n+3} = T_{n+2} + T_{n+1} + T_n$$

| n | TERM | n | TERM | n | TERM | n | TERM |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 41 | 89 | 81 | 176 | 121 | 264 |
| 2 | 3 | 42 | 91 | 82 | 178 | 122 | 266 |
| 3 | 5 | 43 | 93 | 83 | 181 | 123 | 268 |
| 4 | 8 | 44 | 95 | 84 | 183 | 124 | 270 |
| 5 | 10 | 45 | 97 | 85 | 185 | 125 | 272 |
| 6 | 12 | 46 | 99 | 86 | 187 | 126 | 275 |
| 7 | 14 | 47 | 102 | 87 | 189 | 127 | 277 |
| 8 | 16 | 48 | 104 | 88 | 191 | 128 | 279 |
| 9 | 18 | 49 | 106 | 89 | 194 | 129 | 282 |
| 10 | 21 | 50 | 108 | 90 | 196 | 130 | 284 |
| 11 | 23 | 51 | 110 | 91 | 198 | 131 | 286 |
| 12 | 25 | 52 | 113 | 92 | 201 | 132 | 288 |
| 13 | 27 | 53 | 115 | 93 | 203 | 133 | 290 |
| 14 | 29 | 54 | 117 | 94 | 205 | 134 | 292 |
| 15 | 32 | 55 | 119 | 95 | 207 | 135 | 295 |
| 16 | 34 | 56 | 121 | 96 | 209 | 136 | 297 |
| 17 | 36 | 57 | 123 | 97 | 211 | 137 | 299 |
| 18 | 38 | 58 | 126 | 98 | 214 | 138 | 301 |
| 19 | 40 | 59 | 128 | 99 | 216 | 139 | 303 |
| 20 | 42 | 60 | 130 | 100 | 218 | 140 | 306 |
| 21 | 45 | 61 | 133 | 101 | 220 | 141 | 308 |
| 22 | 47 | 62 | 135 | 102 | 222 | 142 | 310 |
| 23 | 49 | 63 | 137 | 103 | 225 | 143 | 312 |
| 24 | 52 | 64 | 139 | 104 | 227 | 144 | 314 |
| 25 | 54 | 65 | 141 | 105 | 229 | 145 | 316 |
| 26 | 56 | 66 | 143 | 106 | 231 | 146 | 319 |
| 27 | 58 | 67 | 146 | 107 | 233 | 147 | 321 |
| 28 | 60 | 68 | 148 | 108 | 235 | 148 | 323 |
| 29 | 62 | 69 | 150 | 109 | 238 | 149 | 326 |
| 30 | 65 | 70 | 152 | 110 | 240 | 150 | 328 |
| 31 | 67 | 71 | 154 | 111 | 242 | 151 | 330 |
| 32 | 69 | 72 | 157 | 112 | 244 | 152 | 332 |
| 33 | 71 | 73 | 159 | 113 | 246 | 153 | 334 |
| 34 | 73 | 74 | 161 | 114 | 248 | 154 | 336 |
| 35 | 76 | 75 | 163 | 115 | 251 | 155 | 339 |
| 36 | 78 | 76 | 165 | 116 | 253 | 156 | 341 |
| 37 | 80 | 77 | 167 | 117 | 255 | 157 | 343 |
| 38 | 82 | 78 | 170 | 118 | 257 | 158 | 345 |
| 39 | 84 | 79 | 172 | 119 | 259 | 159 | 347 |
| 40 | 86 | 80 | 174 | 120 | 262 | 160 | 350 |

## INTEGERS NOT REPRESENTABLE BY THE TRUNCATED QUADRANACCI SEQUENCE

The Quadranacci Sequence is 0,1,1,2,4,8,15,29,56,.....with the recursion relation

$$Q_{n+1} = Q_n + Q_{n-1} + Q_{n-2} + Q_{n-3}$$

The truncated sequence omits the first three terms 0,1,1.

| n | TERM | n | TERM | n | TERM | n | TERM |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 41 | 84 | 81 | 167 | 121 | 250 |
| 2 | 3 | 42 | 86 | 82 | 169 | 122 | 253 |
| 3 | 5 | 43 | 88 | 83 | 171 | 123 | 255 |
| 4 | 7 | 44 | 90 | 84 | 173 | 124 | 257 |
| 5 | 9 | 45 | 92 | 85 | 175 | 125 | 259 |
| 6 | 11 | 46 | 94 | 86 | 177 | 126 | 261 |
| 7 | 13 | 47 | 96 | 87 | 180 | 127 | 263 |
| 8 | 16 | 48 | 98 | 88 | 182 | 128 | 265 |
| 9 | 18 | 49 | 101 | 89 | 184 | 129 | 267 |
| 10 | 20 | 50 | 103 | 90 | 186 | 130 | 269 |
| 11 | 22 | 51 | 105 | 91 | 188 | 131 | 271 |
| 12 | 24 | 52 | 107 | 92 | 190 | 132 | 273 |
| 13 | 26 | 53 | 109 | 93 | 192 | 133 | 275 |
| 14 | 28 | 54 | 111 | 94 | 194 | 134 | 277 |
| 15 | 30 | 55 | 113 | 95 | 196 | 135 | 280 |
| 16 | 32 | 56 | 115 | 96 | 198 | 136 | 282 |
| 17 | 34 | 57 | 117 | 97 | 200 | 137 | 284 |
| 18 | 36 | 58 | 119 | 98 | 202 | 138 | 286 |
| 19 | 38 | 59 | 121 | 99 | 204 | 139 | 288 |
| 20 | 40 | 60 | 124 | 100 | 206 | 140 | 290 |
| 21 | 42 | 61 | 126 | 101 | 209 | 141 | 292 |
| 22 | 45 | 62 | 128 | 102 | 211 | 142 | 294 |
| 23 | 47 | 63 | 130 | 103 | 213 | 143 | 296 |
| 24 | 49 | 64 | 132 | 104 | 215 | 144 | 298 |
| 25 | 51 | 65 | 134 | 105 | 217 | 145 | 300 |
| 26 | 53 | 66 | 136 | 106 | 219 | 146 | 302 |
| 27 | 55 | 67 | 138 | 107 | 221 | 147 | 304 |
| 28 | 57 | 68 | 140 | 108 | 224 | 148 | 306 |
| 29 | 59 | 69 | 142 | 109 | 226 | 149 | 309 |
| 30 | 61 | 70 | 144 | 110 | 228 | 150 | 311 |
| 31 | 63 | 71 | 146 | 111 | 230 | 151 | 313 |
| 32 | 65 | 72 | 148 | 112 | 232 | 152 | 315 |
| 33 | 67 | 73 | 150 | 113 | 234 | 153 | 317 |
| 34 | 69 | 74 | 153 | 114 | 236 | 154 | 319 |
| 35 | 72 | 75 | 155 | 115 | 238 | 155 | 321 |
| 36 | 74 | 76 | 157 | 116 | 240 | 156 | 323 |
| 37 | 76 | 77 | 159 | 117 | 242 | 157 | 325 |
| 38 | 78 | 78 | 161 | 118 | 244 | 158 | 327 |
| 39 | 80 | 79 | 163 | 119 | 246 | 159 | 329 |
| 40 | 82 | 80 | 165 | 120 | 248 | 160 | 332 |

INTEGERS NOT REPRESENTABLE BY THE

SEQUENCE 1,3,5,11,,.WITH $U_{n+2} = U_{n+1} + 2U_n$

| n | TERM | n | TERM | n | TERM | n | TERM |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 41 | 162 | 81 | 322 | 121 | 482 |
| 2 | 7 | 42 | 167 | 82 | 327 | 122 | 487 |
| 3 | 10 | 43 | 170 | 83 | 330 | 123 | 492 |
| 4 | 13 | 44 | 173 | 84 | 333 | 124 | 497 |
| 5 | 18 | 45 | 178 | 85 | 338 | 125 | 500 |
| 6 | 23 | 46 | 181 | 86 | 343 | 126 | 503 |
| 7 | 28 | 47 | 184 | 87 | 348 | 127 | 508 |
| 8 | 31 | 48 | 189 | 88 | 351 | 128 | 511 |
| 9 | 34 | 49 | 194 | 89 | 354 | 129 | 514 |
| 10 | 39 | 50 | 199 | 90 | 359 | 130 | 519 |
| 11 | 42 | 51 | 202 | 91 | 364 | 131 | 522 |
| 12 | 45 | 52 | 205 | 92 | 369 | 132 | 525 |
| 13 | 50 | 53 | 210 | 93 | 372 | 133 | 530 |
| 14 | 53 | 54 | 213 | 94 | 375 | 134 | 535 |
| 15 | 56 | 55 | 216 | 95 | 380 | 135 | 540 |
| 16 | 61 | 56 | 221 | 96 | 383 | 136 | 543 |
| 17 | 66 | 57 | 224 | 97 | 386 | 137 | 546 |
| 18 | 71 | 58 | 227 | 98 | 391 | 138 | 551 |
| 19 | 74 | 59 | 232 | 99 | 394 | 139 | 554 |
| 20 | 77 | 60 | 237 | 100 | 397 | 140 | 557 |
| 21 | 82 | 61 | 242 | 101 | 402 | 141 | 562 |
| 22 | 87 | 62 | 245 | 102 | 407 | 142 | 565 |
| 23 | 92 | 63 | 248 | 103 | 412 | 143 | 568 |
| 24 | 95 | 64 | 253 | 104 | 415 | 144 | 573 |
| 25 | 98 | 65 | 258 | 105 | 418 | 145 | 578 |
| 26 | 103 | 66 | 263 | 106 | 423 | 146 | 583 |
| 27 | 108 | 67 | 266 | 107 | 428 | 147 | 586 |
| 28 | 113 | 68 | 269 | 108 | 433 | 148 | 589 |
| 29 | 116 | 69 | 274 | 109 | 436 | 149 | 594 |
| 30 | 119 | 70 | 279 | 110 | 439 | 150 | 599 |
| 31 | 124 | 71 | 284 | 111 | 444 | 151 | 604 |
| 32 | 127 | 72 | 287 | 112 | 449 | 152 | 607 |
| 33 | 130 | 73 | 290 | 113 | 454 | 153 | 610 |
| 34 | 135 | 74 | 295 | 114 | 457 | 154 | 615 |
| 35 | 138 | 75 | 298 | 115 | 460 | 155 | 620 |
| 36 | 141 | 76 | 301 | 116 | 465 | 156 | 625 |
| 37 | 146 | 77 | 306 | 117 | 468 | 157 | 628 |
| 38 | 151 | 78 | 309 | 118 | 471 | 158 | 631 |
| 39 | 156 | 79 | 312 | 119 | 476 | 159 | 636 |
| 40 | 159 | 80 | 317 | 120 | 479 | 160 | 639 |

INTEGERS NOT REPRESENTABLE BY THE

SEQUENCE 1,3,5,11,....WITH $U_{n+2} = U_{n+1} + 2U_n$

| n | TERM | n | TERM | n | TERM | n | TERM |
|---|---|---|---|---|---|---|---|
| 161 | 642 | 201 | 802 | 241 | 962 | 281 | 1122 |
| 162 | 647 | 202 | 807 | 242 | 967 | 282 | 1127 |
| 163 | 650 | 203 | 810 | 243 | 970 | 283 | 1132 |
| 164 | 653 | 204 | 813 | 244 | 973 | 284 | 1137 |
| 165 | 658 | 205 | 818 | 245 | 978 | 285 | 1140 |
| 166 | 663 | 206 | 821 | 246 | 981 | 286 | 1143 |
| 167 | 668 | 207 | 824 | 247 | 984 | 287 | 1148 |
| 168 | 671 | 208 | 829 | 248 | 989 | 288 | 1151 |
| 169 | 674 | 209 | 834 | 249 | 992 | 289 | 1154 |
| 170 | 679 | 210 | 839 | 250 | 995 | 290 | 1159 |
| 171 | 682 | 211 | 842 | 251 | 1000 | 291 | 1162 |
| 172 | 685 | 212 | 845 | 252 | 1005 | 292 | 1165 |
| 173 | 690 | 213 | 850 | 253 | 1010 | 293 | 1170 |
| 174 | 693 | 214 | 853 | 254 | 1013 | 294 | 1175 |
| 175 | 696 | 215 | 856 | 255 | 1016 | 295 | 1180 |
| 176 | 701 | 216 | 861 | 256 | 1021 | 296 | 1183 |
| 177 | 706 | 217 | 864 | 257 | 1026 | 297 | 1186 |
| 178 | 711 | 218 | 867 | 258 | 1031 | 298 | 1191 |
| 179 | 714 | 219 | 872 | 259 | 1034 | 299 | 1194 |
| 180 | 717 | 220 | 877 | 260 | 1037 | 300 | 1197 |
| 181 | 722 | 221 | 882 | 261 | 1042 | 301 | 1202 |
| 182 | 725 | 222 | 885 | 262 | 1047 | 302 | 1205 |
| 183 | 728 | 223 | 888 | 263 | 1052 | 303 | 1208 |
| 184 | 733 | 224 | 893 | 264 | 1055 | 304 | 1213 |
| 185 | 736 | 225 | 896 | 265 | 1058 | 305 | 1218 |
| 186 | 739 | 226 | 899 | 266 | 1063 | 306 | 1223 |
| 187 | 744 | 227 | 904 | 267 | 1066 | 307 | 1226 |
| 188 | 749 | 228 | 907 | 268 | 1069 | 308 | 1229 |
| 189 | 754 | 229 | 910 | 269 | 1074 | 309 | 1234 |
| 190 | 757 | 230 | 915 | 270 | 1077 | 310 | 1237 |
| 191 | 760 | 231 | 920 | 271 | 1080 | 311 | 1240 |
| 192 | 765 | 232 | 925 | 272 | 1085 | 312 | 1245 |
| 193 | 770 | 233 | 928 | 273 | 1090 | 313 | 1248 |
| 194 | 775 | 234 | 931 | 274 | 1095 | 314 | 1251 |
| 195 | 778 | 235 | 936 | 275 | 1098 | 315 | 1256 |
| 196 | 781 | 236 | 941 | 276 | 1101 | 316 | 1261 |
| 197 | 786 | 237 | 946 | 277 | 1106 | 317 | 1266 |
| 198 | 791 | 238 | 949 | 278 | 1111 | 318 | 1269 |
| 199 | 796 | 239 | 952 | 279 | 1116 | 319 | 1272 |
| 200 | 799 | 240 | 957 | 280 | 1119 | 320 | 1277 |

# FIBONACCI FACTORIALS

The nth Fibonacci factorial is the product of all the Fibonacci numbers from $F_n$ down to $F_1$. These tables present the first 50 factorials in factored form.

| n | FIBONACCI FACTORIAL |
|---|---|
| 1 | 1 |
| 2 | 1 |
| 3 | 2 |
| 4 | $2*3$ |
| 5 | $2*3*5$ |
| 6 | $2^{4}*3*5$ |
| 7 | $2^{4}*3*5*13$ |
| 8 | $2^{4}*3^{2}*5*7*13$ |
| 9 | $2^{5}*3^{2}*5*7*13*17$ |
| 10 | $2^{5}*3^{2}*5^{2}*7*11*13*17$ |
| 11 | $2^{5}*3^{2}*5^{2}*7*11*13*17*89$ |
| 12 | $2^{9}*3^{4}*5^{2}*7*11*13*17*89$ |
| 13 | $2^{9}*3^{4}*5^{2}*7*11*13*17*89*233$ |
| 14 | $2^{9}*3^{4}*5^{2}*7*11*13^{2}*17*29*89*233$ |
| 15 | $2^{10}*3^{4}*5^{3}*7*11*13^{2}*17*29*61*89*233$ |
| 16 | $2^{10}*3^{5}*5^{3}*7^{2}*11*13^{2}*17*29*47*61*89*233$ |
| 17 | $2^{10}*3^{5}*5^{3}*7^{2}*11*13^{2}*17*29*47*61*89*233*1597$ |
| 18 | $2^{13}*3^{5}*5^{3}*7^{2}*11*13^{2}*17^{2}*19*29*47*61*89*233*1597$ |
| 19 | $2^{13}*3^{5}*5^{3}*7^{2}*11*13^{2}*17^{2}*19*29*37*47*61*89*113*233*1597$ |
| 20 | $2^{13}*3^{6}*5^{4}*7^{2}*11^{2}*13^{2}*17^{2}*19*29*37*41*47*61*89*113*233*1597$ |
| 21 | $2^{14}*3^{6}*5^{4}*7^{2}*11^{2}*13^{3}*17^{2}*19*29*37*41*47*61*89*113*233*421*1597$ |
| 22 | $2^{14}*3^{6}*5^{4}*7^{2}*11^{2}*13^{3}*17^{2}*19*29*37*41*47*61*89^{2}*113*199*233*421*1597$ |

# FIBONACCI FACTORIALS

| n | FIBONACCI FACTORIAL |
|---|---|
| 23 | $2^{14}*3^{6}*5^{4}*7^{2}*11^{2}*13^{3}*17^{2}*19*29*37*41*47*61*89^{2}*113*199*233*421*1597*28657$ |
| 24 | $2^{19}*3^{8}*5^{4}*7^{3}*11^{2}*13^{8}*17^{2}*19*23*29*37*41*47*61*89^{2}*113*199*233*421*1597*28657$ |
| 25 | $2^{19}*3^{8}*5^{6}*7^{3}*11^{2}*13^{3}*17^{2}*19*23*29*37*41*47*61*89^{2}*113*199*233*421*1597*3001*28657$ |
| 26 | $2^{19}*3^{8}*5^{6}*7^{3}*11^{2}*13^{3}*17^{2}*19*23*29*37*41*47*61*89^{2}*113*199*233^{2}*421*521*1597*3001*28657$ |
| 27 | $2^{20}*3^{8}*5^{6}*7^{3}*11^{2}*13^{3}*17^{3}*19*23*29*37*41*47*53*61*89^{2}*109*113*199*233^{2}*421*521*1597*3001*28657$ |
| 28 | $2^{20}*3^{9}*5^{6}*7^{3}*11^{2}*13^{4}*17^{3}*19*23*29*37*41*47*53*61*89^{2}*109*113*199*233^{2}*281*421*521*1597*3001*28657$ |
| 29 | $2^{20}*3^{9}*5^{6}*7^{3}*11^{2}*13^{4}*17^{3}*19*23*29^{2}*37*41*47*53*61*89^{2}*109*113*199*233^{2}*281*421*521*1597*3001*28657*514229$ |
| 30 | $2^{23}*3^{9}*5^{7}*7^{3}*11^{3}*13^{4}*17^{3}*19*23*29^{2}*31*37*41*47*53*61^{2}*89^{2}*109*113*199*233^{2}*281*421*521*1597*3001*28657*514229$ |
| 31 | $2^{23}*3^{9}*5^{7}*7^{3}*11^{3}*13^{4}*17^{3}*19*23*29^{2}*31*37*41*47*53*61^{2}*89^{2}*109*113*199*233^{2}*281*421*521*557*1597*2417*3001*28657*514229$ |
| 32 | $2^{23}*3^{10}*5^{7}*7^{4}*11^{3}*13^{4}*17^{3}*19*23*29^{2}*31*37*41*47^{2}*53*61^{2}*89^{2}*109*113*199*233^{2}*281*421*521*557*1597*2207*2417*3001*28657*514229$ |
| 33 | $2^{24}*3^{10}*5^{7}*7^{4}*11^{3}*13^{4}*17^{3}*19*23*29^{2}*31*37*41*47^{2}*53*61^{2}*89^{3}*109*113*199*233^{2}*281*421*521*557*1597*2207*2417*3001*19801*28657*514229$ |
| 34 | $2^{24}*3^{10}*5^{7}*7^{4}*11^{3}*13^{4}*17^{3}*19*23*29^{2}*31*37*41*47^{2}*53*61^{2}*89^{3}*109*113*199*233^{2}*281*421*521*557*1597^{2}*2207*2417*3001*3571*19801*28657*514229$ |

# FIBONACCI FACTORIALS

n FIBONACCI FACTORIAL

35 $2^{24}*3^{10}*5^{8}*7^{4}*11^{3}*13^{5}*17^{3}*19*23*29^{2}*31*37*41*47^{2}*53*61^{2}*89^{3}*109*$
$113*199*233^{2}*281*421*521*557*1597^{2}*2207*2417*3001*3571*19801*$
$28657*141961*514229$

36 $2^{28}*3^{13}*5^{8}*7^{4}*11^{3}*13^{5}*17^{4}*19^{2}*23*29^{2}*31*37*41*47^{2}*53*61^{2}*89^{3}*107*$
$109*113*199*233^{2}*281*421*521*557*1597^{2}*2207*2417*3001*3571*19801*$
$28657*141961*514229$

37 $2^{28}*3^{13}*5^{8}*7^{4}*11^{3}*13^{5}*17^{4}*19^{2}*23*29^{2}*31*37*41*47^{2}*53*61^{2}*73*89^{3}*$
$107*109*113*149*199*233^{2}*281*421*521*557*1597^{2}*2207*2221*2417*3001*$
$3571*19801*28657*141961*514229$

38 $2^{28}*3^{13}*5^{8}*7^{4}*11^{3}*13^{5}*17^{4}*19^{2}*23*29^{2}*31*37^{2}*41*47^{2}*53*61^{2}*73*89^{3}*$
$107*109*113^{2}*149*199*233^{2}*281*421*521*557*1597^{2}*2207*2221*2417*3001*$
$3571*9349*19801*28657*141961*514229$

39 $2^{29}*3^{13}*5^{8}*7^{4}*11^{3}*13^{5}*17^{4}*19^{2}*23*29^{2}*31*37^{2}*41*47^{2}*53*61^{2}*73*89^{3}*$
$107*109*113^{2}*149*199*233^{3}*281*421*521*557*1597^{2}*2207*2221*2417*3001*3571*$
$9349*19801*28657*135721*141961*514229$

40 $2^{29}*3^{14}*5^{9}*7^{5}*11^{4}*13^{5}*17^{4}*19^{2}*23*29^{2}*31*37^{2}*41^{2}*47^{2}*53*61^{2}*73*89^{3}*$
$107*109*113^{2}*149*199*233^{3}*281*421*521*557*1597^{2}*2161*2207*2221*2417*$
$3001*3571*9349*19801*28657*135721*141961*514229$

41 $2^{29}*3^{14}*5^{9}*7^{5}*11^{4}*13^{5}*17^{4}*19^{2}*23*29^{2}*31*37^{2}*41^{2}*47^{2}*53*61^{2}*73*89^{3}*$
$107*109*113^{2}*149*199*233^{3}*281*421*521*557*1597^{2}*2161*2207*2221*2417*$
$2789*3001*3571*9349*19801*28657*59369*135721*141961*514229$

42 $2^{32}*3^{14}*5^{9}*7^{5}*11^{4}*13^{6}*17^{4}*19^{2}*23*29^{3}*31*37^{2}*41^{2}*47^{2}*53*61^{2}*73*89^{3}*$
$107*109*113^{2}*149*199*211*233^{3}*281*421^{2}*521*557*1597^{2}*2161*2207*2221*$
$2417*2789*3001*3571*9349*19801*28657*59369*135721*141961*514229$

43 $2^{32}*3^{14}*5^{9}*7^{5}*11^{4}*13^{6}*17^{4}*19^{2}*23*29^{3}*31*37^{2}*41^{2}*47^{2}*53*61^{2}*73*$
$89^{3}*107*109*113^{2}*149*199*211*233^{3}*281*421^{2}*521*557*1597^{2}*2161*2207*$
$2221*2417*2789*3001*3571*9349*19801*28657*59369*135721*141961*$
$514229*433494437$

44 $2^{32}*3^{15}*5^{9}*7^{5}*11^{4}*13^{6}*17^{4}*19^{2}*23*29^{3}*31*37^{2}*41^{2}*43*47^{2}*53*61^{2}*73*$
$89^{4}*107*109*113^{2}*149*199^{2}*211*233^{3}*281*307*421^{2}*521*557*1597^{2}*2161*$
$2207*2221*2417*2789*3001*3571*9349*19801*28657*59369*135721*141961*$
$514229*433494437$

45 $2^{33}*3^{15}*5^{10}*7^{5}*11^{4}*13^{6}*17^{5}*19^{2}*23*29^{3}*31*37^{2}*41^{2}*43*47^{2}*53*61^{3}*73*$
$89^{4}*107*109*113^{2}*149*199^{2}*211*233^{3}*281*307*421^{2}*521*557*1597^{2}*2161*$
$2207*2221*2417*2789*3001*3571*9349*19801*28657*59369*109441*135721*$
$141961*514229*433494437$

46 $2^{33}*3^{15}*5^{10}*7^{5}*11^{4}*13^{6}*17^{5}*19^{2}*23*29^{3}*31*37^{2}*41^{2}*43*47^{2}*53*61^{3}*73*$
$89^{4}*107*109*113^{2}*139*149*199^{2}*211*233^{3}*281*307*421^{2}*461*521*557*$
$1597^{2}*2161*2207*2221*2417*2789*3001*3571*9349*19801*28657^{2}*59369*$
$109441*135721*141961*514229*433494437$

47 $2^{33}*3^{15}*5^{10}*7^{5}*11^{4}*13^{6}*17^{5}*19^{2}*23*29^{3}*31*37^{2}*41^{2}*43*47^{2}*53*61^{3}*73*$
$89^{4}*107*109*113^{2}*139*149*199^{2}*211*233^{3}*281*307*421^{2}*461*521*557*$
$1597^{2}*2161*2207*2221*2417*2789*3001*3571*9349*19801*28657^{2}*59369*$
$109441*135721*141961*514229*433494437*2971215073$

48 $2^{39}*3^{17}*5^{10}*7^{6}*11^{4}*13^{6}*17^{5}*19^{2}*23^{2}*29^{3}*31*37^{2}*41^{2}*43*47^{3}*53*61^{3}*73*$
$89^{4}*107*109*113^{2}*139*149*199^{2}*211*233^{3}*281*307*421^{2}*461*521*557*1597^{2}*$
$1103*1597^{2}*2161*2207*2221*2417*2789*3001*3571*9349*19801*28657^{2}*$
$59369*109441*135721*141961*514229*433494437*2971215073$

FIBONACCI FACTORIALS

49 $2^{39}*3^{17}*5^{10}*7^{6}*11^{4}*13^{7}*17^{5}*19^{2}*23^{2}*29^{3}*31*37^{2}*41^{2}*43*47^{3}*53*61^{3}*$
$73*89^{4}*97*107*109*113^{2}*139*149*199^{2}*211*233^{3}*281*307*421^{2}*461*521*$
$557*1103*1597^{2}*2161*2207*2221*2417*2789*3001*3571*9349*19801*$
$28657^{2}*59369*109441*135721*141961*514229*6168709*433494437*$
$2971215073$

50 $2^{39}*3^{17}*5^{12}*7^{6}*11^{5}*13^{7}*17^{5}*19^{2}*23^{2}*29^{3}*31*37^{2}*41^{2}*43*47^{3}*53*61^{3}*$
$73*89^{4}*97*101*107*109*113^{2}*139*149*151*199^{2}*211*233^{3}*281*307*421^{2}*$
$461*521*557*1103*1597^{2}*2161*2207*2221*2417*2789*3001^{2}*3571*9349*$
$19801*28657^{2}*59369*109441*135721*141961*514229*6168709*433494437*$
$2971215073$

# FIBONOMIAL COEFFICIENTS

The Fibonomial coefficients are built up from Fibonacci factorials in the same way that binomial coefficients are formed from standard factorials. If $[F_n]!$ represents the nth Fibonacci factorial and F[m,n] a Fibonomial coefficient, then

$$F[m,n] = \frac{[F_m]!}{[F_n]!\,[F_{n-m}]!}$$

$$F[m,0] = F[m,m] = 1 \text{ by definition.}$$

By symmetry $F[m,n] = F[m,m-n]$

In the first part of the table the Fibonomial coefficients and their factorization will be given on the same line. Later when the numbers become large the factorization will be placed below the number.

| | FIBONOMIAL COEFFICIENT | FACTORIZATION |
|---|---|---|
| F[2,1] | 1 | 1 |
| F[3,1] | 2 | 2 |
| F[4,1] | 3 | 3 |
| F[4,2] | 6 | 2*3 |
| F[5,1] | 5 | 5 |
| F[5,2] | 15 | 3*5 |
| F[6,1] | 8 | $2^3$ |
| F[6,2] | 40 | $2^3$*5 |
| F[6,3] | 60 | $2^2$*3*5 |
| F[7,1] | 13 | 13 |
| F[7,2] | 104 | $2^3$*13 |
| F[7,3] | 260 | $2^2$*5*13 |
| F[8,1] | 21 | 3*7 |
| F[8,2] | 273 | 3*7*13 |
| F[8,3] | 1092 | $2^2$*3*7*13 |
| F[8,4] | 1820 | $2^2$*5*7*13 |

FIBONOMIAL COEFFICIENTS

| | FIBONOMIAL COEFFICIENT | FACTORIZATION |
|---|---|---|
| F[9,1] | 34 | $2*17$ |
| F[9,2] | 714 | $2*3*7*17$ |
| F[9,3] | 4641 | $3*7*13*17$ |
| F[9,4] | 12376 | $2^3*7*13*17$ |
| F[10,1] | 55 | $5*11$ |
| F[10,2] | 1870 | $2*5*11*17$ |
| F[10,3] | 19635 | $3*5*7*11*17$ |
| F[10,4] | 85085 | $5*7*11*13*17$ |
| F[10,5] | 136136 | $2^3*7*11*13*17$ |
| F[11,1] | 89 | 89 |
| F[11,2] | 4895 | $5*11*89$ |
| F[11,3] | 83215 | $5*11*17*89$ |
| F[11,4] | 582505 | $5*7*11*17*89$ |
| F[11,5] | 1514513 | $7*11*13*17*89$ |
| F[12,1] | 144 | $2^4*3^2$ |
| F[12,2] | 12816 | $2^4*3^2*89$ |
| F[12,3] | 352440 | $2^3*3^2*5*11*89$ |
| F[12,4] | 3994320 | $2^4*3*5*11*17*89$ |
| F[12,5] | 16776144 | $2^4*3^2*7*11*17*89$ |
| F[12,6] | 27261234 | $2*3^2*7*11*13*17*89$ |
| F[13,1] | 233 | 233 |
| F[13,2] | 33552 | $2^4*3^2*233$ |
| F[13,3] | 1493064 | $2^3*3^2*89*233$ |
| F[13,4] | 27372840 | $2^3*3*5*11*89*233$ |

# FIBONOMIAL COEFFICIENTS

| | FIBONOMIAL COEFFICIENT | FACTORIZATION |
|---|---|---|
| F[13,5] | 186135312 | $2^4*3*11*17*89*233$ |
| F[13,6] | 488605194 | $2*3^2*7*11*17*89*233$ |
| F[14,1] | 377 | $13*29$ |
| F[14,2] | 87841 | $13*29*233$ |
| F[14,3] | 6324552 | $2^3*3^2*13*29*233$ |
| F[14,4] | 187628376 | $2^3*3*13*29*89*233$ |
| F[14,5] | 2063912136 | $2^3*3*11*13*29*89*233$ |
| F[14,6] | 8771626578 | $2*3*11*13*17*29*89*233$ |
| F[14,7] | 1416955062 6 | $2*3^2*7*11*17*29*89*233$ |
| F[15,1] | 610 | $2*5*61$ |
| F[15,2] | 229970 | $2*5*13*29*61$ |
| F[15,3] | 26791505 | $5*13*29*61*233$ |
| F[15,4] | 1285992240 | $2^4*3*5*13*29*61*233$ |
| F[15,5] | 2289066187 2 | $2^4*3*13*29*61*89*233$ |
| F[15,6] | 1573733003 70 | $2*3*5*11*13*29*61*89*233$ |
| F[15,7] | 4115917086 60 | $2^2*3*5*11*17*29*61*89*233$ |
| F[16,1] | 987 | $3*7*47$ |
| F[16,2] | 602070 | $2*3*5*7*47*61$ |
| F[16,3] | 113490195 | $3*5*7*13*29*47*61$ |
| F[16,4] | 8814405145 | $5*7*13*29*47*61*233$ |
| F[16,5] | 2538548681 76 | $2^4*3^2*7*13*29*47*61*233$ |
| F[16,6] | 2824135408 458 | $2*3^2*7*13*29*47*61*89*233$ |
| F[16,7] | 1194826518 9630 | $2*3^2*5*7*11*29*47*61*89*233$ |
| F[16,8] | 1934481030 7020 | $2^2*3*5*11*17*29*47*61*89*233$ |

# FIBONOMIAL COEFFICIENTS

| | FIBONOMIAL COEFFICIENT | FACTORIZATION |
|---|---|---|
| F[17,1] | 1597 | 1597 |
| F[17,2] | 1576239 | 3*7*47*1597 |
| F[17,3] | 480752895 | 3*5*7*47*61*1597 |
| F[17,4] | 6041461380 5 | 5*7*13*29*47*61*1597 |
| F[17,5] | 2815321003 313 | 7*13*29*47*61*233*1597 |
| F[17,6] | 5067577805 9634 | $2*3^{2}*7*13*29*47*61*233*1597$ |
| F[17,7] | 3469341728 69802 | $2*3^{2}*7*29*47*61*89*233*1597$ |
| F[17,8] | 9086371194 20910 | 2*3*5*11*29*47*61*89*233*1597 |
| F[18,1] | 2584 | $2^{3}*17*19$ |
| F[18,2] | 4126648 | $2^{3}*17*19*1597$ |
| F[18,3] | 2036500788 | $2^{2}*3*7*17*19*47*1597$ |
| F[18,4] | 414088935 60 | $2^{3}*5*7*17*19*47*61*1597$ |
| F[18,5] | 3122227241 4424 | $2^{3}*7*13*17*19*29*47*61*1597$ |
| F[18,6] | 9093486840 70099 | 7*13*17*19*29*47*61*233*1597 |
| F[18,7] | 1007278542 3545712 | $2^{4}*3^{2}*7*17*19*29*47*61*233*1597$ |
| F[18,8] | 4268942393 7884208 | $2^{4}*3*17*19*29*47*61*89*233*1597$ |
| F[18,9] | 6905642107 5989160 | $2^{3}*3*5*11*19*29*47*61*89*233*1597$ |
| F[19,1] | 4181 | 37*113 |
| F[19,2] | 10803704 | $2^{3}*17*19*37*113$ |
| F[19,3] | 8626757644 | $2^{2}*17*19*37*113*1597$ |
| F[19,4] | 2838203264 876 | $2^{2}*17*19$ |
| F[19,5] | 346207983 14872 | $2^{3}*7*17*19*37*47*61*113*1597$ |
| F[19,6] | 1631754012 0588343 | 7*13*17*19*29*37*47*61*113*1597 |
| F[19,7] | 2924605267 76698763 | 7*17*19*29*37*47*61*113*233*1597 |

| | | |
|---|---|---|
| F[19,8] | 2005443612 183077232 | $2^4$*3*17*19*29*37*47*61*113*233*1597 |
| F[19,9] | 5249543573 067466872 | $2^3$*3*19*29*37*47*61*89*113*233*1597 |
| F[20,1] | 6765 | 3*5*11*41 |
| F[20,2] | 28284465 | 3*5*11*37*41*113 |
| F[20,3] | 3654352878 0 | $2^2$*3*5*11*17*19*37*41*113 |
| F[20,4] | 1945333848 7220 | $2^2$*5*11*17*19*37*41*113*1597 |
| F[20,5] | 3840089017 377228 | $2^2$*3*7*11*17*19*37*41*47*113*1597 |
| F[20,6] | 2928067875 75013635 | 3*5*7*11*17*19*37*41*47*61*113*1597 |
| F[20,7] | 8491396839 675395415 | 3*5*7*11*17*19*29*37*41*47*61*113*1597 |
| F[20,8] | 9421406969 7350815795 | 5*11*17*19*29*37*41*47*61*113*233*1597 |
| F[20,9] | 3990242951 8877992572 0 | $2^3$*$3^2$*5*11*19*29*37*41*47*61*113*233*1597 |
| F[20,10] | 6456938594 8729842525 6 | $2^3$*$3^2$*19*29*37*41*47*61*89*113*233*1597 |
| F[21,1] | 10946 | 2*13*421 |
| F[21,2] | 74049690 | 2*3*5*11*13*41*421 |
| F[21,3] | 1548008769 45 | 3*5*11*13*37*41*113*421 |
| F[21,4] | 1333351553 41960 | $2^3$*5*11*13*17*19*37*41*113*421 |
| F[21,5] | 4258724861 6222024 | $2^3$*11*13*17*19*37*41*113*421*1597 |
| F[21,6] | 5254201798 026392211 | 3*7*11*13*17*19*37*41*47*113*421*1597 |
| F[21,7] | 2465433151 3816148067 0 | 2*3*5*7*11*17*19*37*41*47*61*113*421*1597 |
| F[21,8] | 4426039514 6231846767 90 | 2*5*11*13*17*19*29*37*41*47*61*113*421*1597 |
| F[21,9] | 3033138843 8447118520 355 | 5*11*13*19*29*37*41*47*61*113*233*421*1597 |
| F[21,10] | 7941308972 9752455762 384 | $2^4$*$3^2$*13*19*29*37*41*47*61*113*233*421*1597 |
| F[22,1] | 17711 | 89*199 |
| F[22,2] | 193864606 | 2*13*89*199*421 |
| F[22,3] | 6557470297 95 | 3*5*11*13*41*89*199*421 |

F[22,4] 9138927771 90965

5*11*13*37*41*89*113*199*421

F[22,5] 4722997872 52290712

$2^3$*11*13*17*19*37*41*89*113*199*421

F[22,6] 9428284503 0238533383

11*13*17*19*37*41*89*113*199*421*1597

F[22,7] 7158243695 7573409576 17

3*7*11*17*19*37*41*47*89*113*199*421*1597

F[22,8] 2079299359 2437990400 6970

2*5*11*17*19*37*41*47*61*89*113*199*421*1597

F[22,9] 2305576054 2203301120 77285

5*11*13*19*29*37*41*47*61*89*113*199*421*1597

F[22,10] 9767258556 9697621111 63771

13*19*29*37*41*47*61*89*113*199*233*421*1597

F[22,11] 1580320485 6220738696 714416

$2^4$*$3^2$*13*19*29*37*41*47*61*113*199*233*421*1597

F[23,1] 28657

28657

F[23,2] 507544127

89*199*28657

F[23,3] 2777789007 071

13*89*199*421*28657

F[23,4] 6263914210 945105

5*11*13*41*89*199*421*28657

F[23,5] 5237885063 192296801

11*13*37*41*89*113*199*421*28657

F[23,6] 1691836875 4111118667 23

11*13*17*19*37*41*89*113*199*421*28675

F[23,7] 2078356530 7934966547 3587

11*17*19*37*41*89*113*199*421*1597*28657

F[23,8] 9768275694 7294342772 58589

11*17*19*37*41*47*89*113*199*421*1597*28657

F[23,9] 1752543580 5249867379 7874685

5*11*19*37*41*47*61*89*113*199*421*1597*28657

F[23,10] 1201288963 3780363640 32704659

13*19*29*37*41*47*61*89*113*199*421*1597*28657

F[23,11] 3144947510 8660952002 20451523

13*19*29*37*41*47*61*113*199*233*421*1597*28657

F[24,1] 46368

$2^5*3^2*7*23$

F[24,2] 1328767776

$2^5*3^2*7*23*28657$

F[24,3] 1176690304 0368

$2^4*3^2*7*23*89*199*28657$

F[24,4] 4293350689 3289376

$2^5*3*7*13*23*89*199*421*28657$

F[24,5] 5808903482 6620525728

$2^5*3^2*7*11*13*23*41*89*199*421*28657$

F[24,6] 3035878182 6262552258 596

$2^2*3^2*7*11*13*23*37*41*89*113*199*421*28657$

F[24,7] 6034391710 6971103874 00928

$2^5*3^2*7*11*17*19*23*37*41*89*113*199*421*28657$

F[24,8] 4589011219 9920406136 5680096

$2^5*3*11*17*19*23*37*41*89*113*199*421*1597*28657$

F[24,9] 1332162962 9800423781 409595728

$2^4*3^2*7*11*19*23*37*41*47*89*113*199*421*1597*28657$

F[24,10] 1477489831 6687742739 3815516256

$2^5*3^2*7*19*23*37*41*47*61*89*113*199*421*1597*28657$

F[24,11] 6258580522 9115494525 2454490208

$2^5*3^2*7*13*19*23*29*37*41*47*61*113*199*421*1597*28657$

F[24,12] 1012673098 4988826544 7098539040 6

2*7*13*19*23*29*37*41*47*61*113*199*233*421*1597*28657

F[25,1] 75025

$5^2*3001$

F[25,2] 3478759200

$2^5*3^2*5^2*7*23*3001$

F[25,3] 4984540119 7200

$2^4*3^2*5^2*7*23*3001*28657$

F[25,4] 2942706335 34536400

$2^4*3*5^2*7*23*89*199*3001*28657$

F[25,5] 6442172709 3380708688 0

$2^5*3*5*7*13*23*89*199*421*3001*28657$

F[25,6] 5447662297 3340061784 2900

$2^2*3^2*5^2*7*11*13*23*41*89*199*421*3001*28675$

F[25,7] 1752052005 0118061409 2397300

$2^2*3^2*5^2*7*11*23*37*41*89*113*199*421*3001*28657$

F[25,8] 2155858276 6430986038 797839200

$2^5*3*5^2*11*17*19*23*37*41*89*113*199*421*3001*28657$

F[25,9] 1012619314 0585377854 1059262360 0

$2^4*3*5^2*11*19*23*37*41*89*113*199*421*1597*3001*28657$

F[25,10] 1817191387 2286850803 6409985362 40

$2^4*3^2*5*7*19*23*37*41*47*89*113*199*421*1597*3001*28657$

F[25,11] 1245490726 0780875269 9112461877 600

$2^5*3^2*5^2*7*19*23*37*41*47*61*113*199*421*1597*3001*28657$

F[25,12] 3260763914 8016597060 8093042554 550

$2*5^2*7*13*19*23*29*37*41*47*61*113*199*421*1597*3001*28657$

F[26,1] 121393

233*521

F[26,2] 9107509825

$5^2*233*521*3001$

F[26,3] 2111485077 82800

$2^4*3^2*5^2*7*23*233*521*3001$

F[26,4] 2016960929 177233200

$2^4*3*5^2*7*23*233*521*3001*28657$

F[26,5] 7144479003 3315954410 40

$2^4*3*5*7*23*89*199*233*521*3001*28657$

F[26,6] 9775433396 3084554622 02980

$2^2*3*5*7*13*23*89*199*233*421*521*3001*28657$

F[26,7] 5086985148 1559000924 46396900

$2^2*3^2*5^2*7*11*23*41*89*199*233*421*521*3001*28657$

F[26,8] 1012794519 2590389660 2468502090 0

$2^2*3*5^2*11*23*37*41*89*113*199*233*421*521*3001*28657$

F[26,9] 7697238346 3686961417 8760615884 00

$2^4*3*5^2*11*19*23*37*41*89*113*199*233*421*521*3001*28657$

F[26,10] 2234998116 2092377706 2451037012 1360

$2^4*3*5*19*23*37*41*89*113*199*233*421*521*1597*3001*28657$

F[26,11] 2478587798 5376603141 6448509585 36880

$2^4*3^2*5*7*19*23*37*41*47*113*199*233*421*521*1597*3001*28657$

F[26,12] 1049957331 3249811053 0578882532 690950

$2*5^2*7*19*23*37*41*47*61*113*199*233*421*521*1597*3001*28657$

F[26,13] 1698857999 6116647068 6816475170 920550

$2*5^2*7*13*19*23*29*37*41*47*61*113*199*421*521*1597*3001*28657$

F[27,1] 196418

2*17*53*109

F[27,2] 2384377027 4

2*17*53*109*233*521

F[27,3] 8944394324 03425

$5^2*17*53*109*233*521*3001$

F[27,4] 1382445586 7227336800

$2^5*3*5^2*7*17*23*53*109*233*521*3001$

F[27,5] 7923348635 7426758135 520

$2^5*3*5*7*17*23*53*109*233*521*3001*28657$

F[27,6] 1754130346 0954816416 7274340

$2^2*3*5*7*17*23*53*89*109*199*233*521*3001*28657$

F[27,7] 1476977751 4123955422 8844994280

$2^3*3*5*7*17*23*53*89*109*199*233*421*521*3001*28657$

F[27,8] 4757978327 7642170683 7207803002 00

$2^3*3*5^2*11*17*23*41*53*89*109*199*233*421*521*3001*28657$

F[27,9] 5850913937 7594681067 2460536573 9300

$2^2*3*5^2*11*23*37*41*53*89*109*113*199*233*421*521*3001*28657$

F[27,10] 2748865748 2128119250 5025095728 551840

$2^5*3*5*17*19*23*37*41*53*89*109*113*199*233*421*521*3001*28657$

F[27,11] 4932515280 7818658924 7780986380 3340320

$2^5*3*5*17*19*23*37*41*53*109*113*199*233*421*521*1597*3001*28657$

F[27,12] 3380828182 0359039138 0249884414 85395110

2*5*7*17*19*23*37*41*47*53*109*113*199*233*421*521*1597*
3001*28657

F[27,13] 8851095240 5231819202 5546907695 54038700

$2^2*5^2*7*17*19*23*37*41*47*53*61*109*113*199*421*521*1597*$
3001*28657

F[28,1] 317811

3*13*29*281

F[28,2] 6242380099 8

2*3*13*17*29*53*109*281

F[28,3] 3788906237 275107

3*13*17*29*53*109*233*281*521

F[28,4] 9475423015 0521634225

$5^2*13*17*29*53*109*233*281*521*3001$

F[28,5] 8787128287 2387742714 8960

$2^5*3^2*5*7*13*17*23*29*53*109*233*281*521*3001$

F[28,6] 3147659191 5925194287 25968340

$2^2*3^2*5*7*13*17*23*29*53*109*233*281*521*3001*28657$

F[28,7] 4288322457 0996239693 9735578998 0

$2^2*3^2*5*7*17*23*29*53*89*109*199*233*281*521*3001*28657$

F[28,8] 2235237029 3053563794 7730745129 1480

$2^3$*3*5*13*17*23*29*53*89*109*199*233*281*421*521*3001*28657

F[28,9] 4447464265 6619811491 6587791411 378300

$2^2$*$3^2$*$5^2$*11*13*23*29*41*53*89*109*199*233*281*421*521*3001*28657

F[28,10] 3380881471 7695896699 3864282889 26775860

$2^2$*$3^2$*5*13*23*29*37*41*53*89*109*113*199*233*281*421*521*
3001*28657

F[28,11] 9815952497 8119322551 9264389871 761672160

$2^5$*$3^2$*5*13*17*19*23*29*37*41*53*109*113*199*233*281*421*521*
3001*28657

F[28,12] 1088616398 5420594313 5712863237 8613465583 0

2*5*13*17*19*23*29*37*41*53*109*113*199*233*281*421*521*
1597*3001*28657

F[28,13] 4611435130 3047753594 3985390625 6186654637 0

2*3*5*7*13*17*19*23*29*37*41*47*53*109*113*199*281*421*521*
1597*3001*28657

F[28,14] 7461473287 7610423587 7536043187 3405462410 0

$2^2$*3*$5^2$*7*17*19*23*37*41*47*53*61*109*113*199*281*421*521*
1597*3001*28657

F[29,1] 514229

514229

F[29,2] 1634276327 19

3*13*29*281*514229

F[29,3] 1605006438 1700271

3*13*17*29*53*109*281*514229

F[29,4] 6494551551 6258033250 1

13*17*29*53*109*233*281*521*514229

F[29,5] 9745074603 2145178891 77505

5*13*17*29*53*109*233*281*521*3001*514229

F[29,6] 5648245240 0231345685 672818980

$2^2*3^2*5*7*13*17*23*29*53*109*233*281*521*3001*514229$

F[29,7] 1245090491 1026382102 4178921039 220

$2^2*3^2*5*7*17*23*29*53*109*233*281*521*3001*28657*514229$

F[29,8] 1050085604 1866107305 5201565263 125020

$2^2*3*5*17*23*29*53*89*109*199*233*281*521*3001*28657*514229$

F[29,9] 3380657948 0666591343 0069509814 75484380

$2^2*3*5*13*23*29*53*89*109*199*233*281*421*521*3001*28657*$
514229

F[29,10] 4158209276 1219907351 8985497072 1484578740

$2^2*3^2*5*13*23*29*41*53*89*109*199*233*281*421*521*3001*$
28657*514229

F[29,11] 1953423930 7265217150 3694198118 9497418394 60

$2^2*3^2*5*13*23*29*37*41*53*109*113*199*233*281*421*521*$
3001*28657*514229

F[29,12] 3505310720 1370361886 4962366624 5598145230 310

2*5*13*17*19*23*29*37*41*53*109*113*199*233*281*421*521*
3001*28657*514229

F[29,13] 2402567047 2355565636 3667338840 9536582803 7790

2*5*13*17*19*23*29*37*41*53*109*113*199*281*421*521*1597
3001*28657*514229

F[29,14] 6290009749 6591361493 6179478610 1359700868 2490

2*3*5*7*17*19*23*37*41*47*53*109*113*199*281*421*521*1597*
3001*28657*514229

F[30,1] 832040

$2^3*5*11*31*61$

F[30,2] 4278590971 60

$2^3*5*11*31*61*514229$

F[30,3] 6798916376 3758380

$2^2*3*5*11*13*29*31*61*281*514229$

## FIBONOMIAL COEFFICIENTS

F[30,4] 4451431856 0499644942 80

$2^3$*5*11*13*17*29*31*53*61*109*281*514229

F[30,5] 1080745334 6029466797 0826408

$2^3$*11*13*17*29*31*53*61*109*233*281*521*514229

F[30,6] 1013536484 1073259330 6390640752 5

$5^2$*11*13*17*29*31*53*61*109*233*281*521*3001*514229

F[30,7] 3615050745 7760376049 4670863877 8400

$2^5$*$3^2$*$5^2$*7*11*17*23*29*31*53*61*109*233*281*521*3001*514229

F[30,8] 4933167105 7954242688 0751568864 1552800

$2^5$*3*$5^2$*11*17*23*29*31*53*61*109*233*281*521*3001*28657*
514229

F[30,9] 2569744782 6689046830 8382089298 6192401200

$2^4$*3*$5^2$*11*23*29*31*53*61*89*109*199*233*281*521*3001*28657*
514229

F[30,10] 5114259343 8352419383 7009154447 7611277006 40

$2^5$*3*5*13*23*29*31*53*61*89*109*199*233*281*421*521*3001*
28657*514229

F[30,11] 3887411737 1961136756 2625497734 7307897634 6400

$2^5$*$3^2$*$5^2$*11*13*23*29*31*41*53*61*109*199*233*281*421*521*
3001*28657*514229

F[30,12] 1128699199 5289549498 4676194863 1187105556 279850

2*$5^2$*11*13*23*29*31*37*41*53*61*109*113*199*233*281*421*
521*3001*28657*514229

F[30,13] 1251741944 8853302963 1074372328 8835828651 2562800

$2^4$*$5^2$*11*13*17*19*23*29*31*37*41*53*61*109*113*199*281*421*
521*3001*28657*514229

F[30,14] 5302471846 1057625549 2906558645 1646733039 9370800

$2^4$*$5^2$*11*17*19*23*31*37*41*53*61*109*113*199*281*421*521*
1597*3001*28657*514229

F[30,15] 8579573298 5350617077 2948808824 2254631984 2916360

$2^3$*3*5*7*11*17*19*23*31*37*41*47*53*109*113*199*281*421*
521*1597*3001*28657*514229

## TABLES OF FIBONOMIAL COEFFICIENTS

Fibonomial coefficients formed from the terms of the sequence $F_k$ (1,1,2,3, 5,8,13,21,34,.....)

| n | FIBONOMIAL COEFFICIENTS(k=1) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | | | | | | |
| 2 | 1 | 1 | 1 | | | | | | | | |
| 3 | 1 | 2 | 2 | 1 | | | | | | | |
| 4 | 1 | 3 | 6 | 3 | 1 | | | | | | |
| 5 | 1 | 5 | 15 | 15 | 5 | 1 | | | | | |
| 6 | 1 | 8 | 40 | 60 | 40 | 8 | 1 | | | | |
| 7 | 1 | 13 | 104 | 260 | 260 | 104 | 13 | 1 | | | |
| 8 | 1 | 21 | 273 | 1092 | 1820 | 1092 | 273 | 21 | 1 | | |
| 9 | 1 | 34 | 714 | 4641 | 12376 | 12376 | 4641 | 714 | 34 | 1 | |
| 10 | 1 | 55 | 1870 | 19635 | 85085 | 136136 | 85085 | 19635 | 1870 | 55 | 1 |

Fibonomial coefficients formed from the terms of the sequence $F_{2k}$(1,3,8,21, 55,144,377,987,......)

| n | FIBONOMIAL COEFFICIENTS(k=2) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | | | | | | |
| 2 | 1 | 3 | 1 | | | | | | | | |
| 3 | 1 | 8 | 8 | 1 | | | | | | | |
| 4 | 1 | 21 | 56 | 21 | 1 | | | | | | |
| 5 | 1 | 55 | 385 | 385 | 55 | 1 | | | | | |
| 6 | 1 | 144 | 2640 | 6930 | 2640 | 144 | 1 | | | | |
| 7 | 1 | 377 | 18096 | 124410 | 124410 | 18096 | 377 | 1 | | | |
| 8 | 1 | 987 | 124033 | 2232594 | 5847270 | 2232594 | 124033 | 987 | 1 | | |
| 9 | 1 | 2584 | 850136 | 40062659 | 274715376 | 274715376 | 40062659 | 850136 | 2584 | 1 | |
| 10 | 1 | 6765 | 5826920 | 718896255 | 12905899435 | 33789991248 | 12905899435 | 718896255 | 5826920 | 6765 | 1 |

Fibonomial coefficients formed from the terms of the sequence $F_{3k}$(2,8,34, 144,610,2584,10946,46368,196418,832040,.....)

| n | FIBONOMIAL COEFFICIENTS (k=3) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | | | | | | |
| 2 | 1 | 4 | 1 | | | | | | | | |
| 3 | 1 | 17 | 17 | 1 | | | | | | | |
| 4 | 1 | 72 | 306 | 72 | 1 | | | | | | |
| 5 | 1 | 305 | 5490 | 5490 | 305 | 1 | | | | | |
| 6 | 1 | 1292 | 98515 | 417240 | 98515 | 1292 | 1 | | | | |
| 7 | 1 | 5473 | 1767779 | 31716035 | 31716035 | 1767779 | 5473 | 1 | | | |
| 8 | 1 | 23184 | 31721508 | 2410834608 | 10212563270 | 2410834608 | 31721508 | 23184 | 1 | | |
| 9 | 1 | 98209 | 569219364 | 183255151716 | 3288411889126 | 3288411889126 | 183255151716 | 569219364 | 98209 | 1 | |
| 10 | 1 | 416020 | 10214227045 | 13929802341840 | 1058858447456810 | 4485393816767864 | 1058858447456810 | 13929802341840 | 10214227045 | 416020 | 1 |

# TABLES OF FIBONOMIAL COEFFICIENTS

Fibonomial coefficients formed from the terms of the sequence $F_{4k}$(3,21,144, 987,6765,46368,317811, 2178309, 14930352, 102334155,......)

| n | FIBONOMIAL COEFFICIENTS | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | | | | | | |
| 2 | 1 | 7 | 1 | | | | | | | | |
| 3 | 1 | 48 | 48 | 1 | | | | | | | |
| 4 | 1 | 329 | 2256 | 329 | 1 | | | | | | |
| 5 | 1 | 2255 | 105985 | 105985 | 2255 | 1 | | | | | |
| 6 | 1 | 15456 | 4979040 | 34127170 | 4979040 | 15456 | 1 | | | | |
| 7 | 1 | 105937 | 233908896 | 10988845010 | 10988845010 | 233908896 | 105937 | 1 | | | |
| 8 | 1 | 726103 | 10988739073 | 3538373981506 | 24252380937070 | 3538373981506 | 10988739073 | 726103 | 1 | | |
| 9 | 1 | 4976784 | 516236827536 | 1139345433305859 | 53524993973177376 | 53524993973177376 | 1139345433305859 | 516236827536 | 4976784 | 1 | |
| 10 | 1 | 34111385 | 24252142155120 | 366865691151231195 | 11812963745741027O835 | 8096725838322541667 52 | 118129637457410270835 | 366865691151231195 | 24252142155120 | 34111385 | 1 |

Fibonomial coefficients formed from the terms of the sequence $F_{5k}$(5,55,610, 6765, 75025, 832040, 9227465, 102334155,......)

| n | FIBONOMIAL COEFFICIENTS | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | | | | |
| 2 | 1 | 11 | 1 | | | | | | |
| 3 | 1 | 122 | 122 | 1 | | | | | |
| 4 | 1 | 1353 | 15006 | 1353 | 1 | | | | |
| 5 | 1 | 15005 | 1845615 | 1845615 | 15005 | 1 | | | |
| 6 | 1 | 166408 | 226995640 | 2517418860 | 226995640 | 166408 | 1 | | |
| 7 | 1 | 1845493 | 27918618104 | 3433761185660 | 3433761185660 | 27918618104 | 1845493 | 1 | |
| 8 | 1 | 20466831 | 3433763031153 | 4683652774492692 | 51942505455478820 | 4683652774492692 | 3433763031153 | 20466831 | 1 |

Fibonomial coefficients formed from the terms of the sequence $F_{6k}$(8,144, 2584, 46368, 832040, 14930352, 267914296,......)

| n | FIBONOMIAL COEFFICIENTS | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | | | |
| 2 | 1 | 18 | 1 | | | | | |
| 3 | 1 | 323 | 323 | 1 | | | | |
| 4 | 1 | 5796 | 104006 | 5796 | 1 | | | |
| 5 | 1 | 104005 | 33489610 | 33489610 | 104005 | 1 | | |
| 6 | 1 | 1866294 | 10783550415 | 193502966580 | 10783550415 | 1866294 | 1 | |
| 7 | 1 | 33489287 | 3472269744021 | 1118060107513635 | 1118060107513635 | 3472269744021 | 33489287 | 1 |

## TABLES OF FIBONOMIAL COEFFICIENTS

Fibonomial coefficients formed from the terms of the sequence $F_{7k}$(13,377, 10946, 317811, 9227465, 267914296,.....)

| n | FIBONOMIAL COEFFICIENTS | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | | |
| 2 | 1 | 29 | 1 | | | | |
| 3 | 1 | 842 | 842 | 1 | | | |
| 4 | 1 | 24447 | 709806 | 24447 | 1 | | |
| 5 | 1 | 709805 | 598365615 | 598365615 | 709805 | 1 | |
| 6 | 1 | 20608792 | 504421503640 | 14645596792740 | 504421503640 | 20608792 | 1 |

Fibonomial coefficients formed from the terms of the sequence $F_{8k}$(21,987, 46368, 2178309, 102334155, 4807526976, .....)

| n | FIBONOMIAL COEFFICIENTS | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | | |
| 2 | 1 | 47 | 1 | | | | |
| 3 | 1 | 2208 | 2208 | 1 | | | |
| 4 | 1 | 103729 | 4873056 | 103729 | 1 | | |
| 5 | 1 | 4873055 | 10754832385 | 10754832385 | 4873055 | 1 | |
| 6 | 1 | 228929856 | 23735910200640 | 1115082531341570 | 23735910200640 | 228929856 | 1 |

Fibonomial coefficients formed from the terms of the sequence $F_{9k}$(34,2584, 196418, 14930352, 1134903170, 86267571272,.....)

| n | FIBONOMIAL COEFFICIENTS | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | |
| 2 | 1 | 76 | 1 | | | |
| 3 | 1 | 5777 | 5777 | 1 | | |
| 4 | 1 | 439128 | 33379506 | 439128 | 1 | |
| 5 | 1 | 33379505 | 192866779890 | 192866779890 | 33379505 | 1 |

Fibonomial coefficients formed from the terms of the sequence $F_{10k}$(55,6765, 832040, 102334155, 12586269025,.....)

| n | FIBONOMIAL COEFFICIENTS | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | |
| 2 | 1 | 123 | 1 | | | |
| 3 | 1 | 15128 | 15128 | 1 | | |
| 4 | 1 | 1860621 | 228841256 | 1860621 | 1 | |
| 5 | 1 | 228841255 | 3461681664385 | 3461681664385 | 228841255 | 1 |

## FIBONACCI POLYNOMIAL COEFFICIENTS

If we define $f_1(x) = 1$, $f_2(x) = x$, and $f_{n+2}(x) = xf_{n+1}(x) + f_n(x)$, the coefficients of the resulting polynomials are as given in the table going from the highest to the lowest power, the highest power being equivalent to n and the powers descending by two at each step.

| n | Coefficients | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | | | | | | | | | | |
| 1 | 1 | | | | | | | | | | | | | | | |
| 2 | 1 | 1 | | | | | | | | | | | | | | |
| 3 | 1 | 2 | | | | | | | | | | | | | | |
| 4 | 1 | 3 | 1 | | | | | | | | | | | | | |
| 5 | 1 | 4 | 3 | | | | | | | | | | | | | |
| 6 | 1 | 5 | 6 | 1 | | | | | | | | | | | | |
| 7 | 1 | 6 | 10 | 4 | | | | | | | | | | | | |
| 8 | 1 | 7 | 15 | 10 | 1 | | | | | | | | | | | |
| 9 | 1 | 8 | 21 | 20 | 5 | | | | | | | | | | | |
| 10 | 1 | 9 | 28 | 35 | 15 | 1 | | | | | | | | | | |
| 11 | 1 | 10 | 36 | 56 | 35 | 6 | | | | | | | | | | |
| 12 | 1 | 11 | 45 | 84 | 70 | 21 | 1 | | | | | | | | | |
| 13 | 1 | 12 | 55 | 120 | 126 | 56 | 7 | | | | | | | | | |
| 14 | 1 | 13 | 66 | 165 | 210 | 126 | 28 | 1 | | | | | | | | |
| 15 | 1 | 14 | 78 | 220 | 330 | 252 | 84 | 8 | | | | | | | | |
| 16 | 1 | 15 | 91 | 286 | 495 | 462 | 210 | 36 | 1 | | | | | | | |
| 17 | 1 | 16 | 105 | 364 | 715 | 792 | 462 | 120 | 9 | | | | | | | |
| 18 | 1 | 17 | 120 | 455 | 1001 | 1287 | 924 | 330 | 45 | 1 | | | | | | |
| 19 | 1 | 18 | 136 | 560 | 1365 | 2002 | 1716 | 792 | 165 | 10 | | | | | | |
| 20 | 1 | 19 | 153 | 680 | 1820 | 3003 | 3003 | 1716 | 495 | 55 | 1 | | | | | |
| 21 | 1 | 20 | 171 | 816 | 2380 | 4368 | 5005 | 3432 | 1287 | 220 | 11 | | | | | |
| 22 | 1 | 21 | 190 | 969 | 3060 | 6188 | 8008 | 6435 | 3003 | 715 | 66 | 1 | | | | |
| 23 | 1 | 22 | 210 | 1140 | 3876 | 8568 | 12376 | 11440 | 6435 | 2002 | 286 | 12 | | | | |
| 24 | 1 | 23 | 231 | 1330 | 4845 | 11628 | 18564 | 19448 | 12870 | 5005 | 1001 | 78 | 1 | | | |
| 25 | 1 | 24 | 253 | 1540 | 5985 | 15504 | 27132 | 31824 | 24310 | 11440 | 3003 | 364 | 13 | | | |
| 26 | 1 | 25 | 276 | 1771 | 7315 | 20349 | 38760 | 50388 | 43758 | 24310 | 8008 | 1365 | 91 | 1 | | |
| 27 | 1 | 26 | 300 | 2024 | 8855 | 26334 | 54264 | 77520 | 75582 | 48620 | 19448 | 4368 | 455 | 14 | | |
| 28 | 1 | 27 | 325 | 2300 | 10626 | 33649 | 74613 | 116280 | 125970 | 92378 | 43758 | 12376 | 1820 | 105 | 1 | |
| 29 | 1 | 28 | 351 | 2600 | 12650 | 42504 | 100947 | 170544 | 203490 | 167960 | 92378 | 31824 | 6188 | 560 | 15 | |
| 30 | 1 | 29 | 378 | 2925 | 14950 | 53130 | 134596 | 245157 | 319770 | 293930 | 184756 | 75582 | 18564 | 2380 | 120 | 1 |

## CONTINUED FRACTION QUOTIENTS OF LINEAR FIBONACCI RATIOS

### $F_n/F_{n-1}$

| n | QUOTIENTS |
|---|---|
| 2 | (1) |
| 3 | (2) |
| 4 | (1,2) |
| 5 | (1,1,2) |
| 6 | (1,1,1,2) |
| n | $(1_{n-3},2)$ |

### $F_n/F_{n-2}$

| n | QUOTIENTS |
|---|---|
| 3 | (2) |
| 4 | (3) |
| 5 | (2,2) |
| 6 | (2,1,2) |
| 7 | (2,1,1,2) |
| n | $(2,1_{n-5},2)$ |

### $F_n/F_{n-3}$

| n | QUOTIENTS |
|---|---|
| 4 | (3) |
| 5 | (5) |
| 6 | (4) |
| 7 | (4,3) |
| 8 | (4,5) |
| 9 | (4,4) |
| 10 | (4,4,3) |
| 11 | (4,4,5) |
| 12 | (4,4,4) |
| 3k | $(4_{k-1})$ |
| 3k+1 | $(4_{k-1},3)$ |
| 3k+2 | $(4_{k-1},5)$ |

### $F_n/F_{n-4}$

| n | QUOTIENTS |
|---|---|
| 5 | (5) |
| 6 | (8) |
| 7 | (6,2) |
| 8 | (7) |
| 9 | (6,1,4) |
| 10 | (6,1,7) |
| 11 | (6,1,5,2) |
| 12 | (6,1,6) |
| 13 | (6,1,5,1,4) |
| 14 | (6,1,5,1,7) |
| 15 | (6,1,5,1,5,2) |
| 16 | (6,1,5,1,6) |
| 17 | (6,1,5,1,5,1,4) |
| 18 | (6,1,5,1,5,1,7) |
| 19 | (6,1,5,1,5,1,5,2) |
| 4k | $(6, (1,5)_{k-3}, 1,6)$ |
| 4k+1 | $(6, (1,5)_{k-2}, 1,4)$ |
| 4k+2 | $(6, (1,5)_{k-2}, 1,7)$ |
| 4k+3 | $(6, (1,5)_{k-1}, 2)$ |

### $F_n/F_{n-5}$

| n | QUOTIENTS |
|---|---|
| 6 | (8) |
| 7 | (13) |
| 8 | (10,2) |
| 9 | (11,3) |
| 10 | (11) |
| 11 | (11,8) |
| 12 | (11,13) |
| 13 | (11,10,2) |
| 14 | (11,11,3) |
| 15 | (11,11) |
| 5k | $(11_{k-1})$ |
| 5k+1 | $(11_{k-1},8)$ |
| 5k+2 | $(11_{k-1},13)$ |
| 5k+3 | $(11_{k-1},10,2)$ |
| 5k+4 | $(11_k, 3)$ |

## CONTINUED FRACTION QUOTIENTS OF LINEAR FIBONACCI RATIOS

### $F_n/F_{n-6}$

| n | QUOTIENTS |
|---|---|
| 7 | (13) |
| 8 | (21) |
| 9 | (17) |
| 10 | (18,3) |
| 11 | (17,1,4) |
| 12 | (18) |
| 13 | (17,1,12) |
| 14 | (17,1,20) |
| 15 | (17,1,16) |
| 16 | (17,1,17,3) |
| 17 | (17,1,16,1,4) |
| 18 | (17,1,17) |
| 19 | (17,1,16,1,12) |
| 20 | (17,1,16,1,20) |
| 21 | (17,1,16,1,16) |
| 22 | (17,1,16,1,17,3) |
| 23 | (17,1,16,1,16,1,4) |
| 24 | (17,1,16,1,17) |
| 6k | $(17,\ (1,16)_{k-3},\ 1,17)$ |
| 6k+1 | $(17,\ (1,16)_{k-2},\ 1,12)$ |
| 6k+2 | $(17,\ (1,16)_{k-2},\ 1,20)$ |
| 6k+3 | $(17,\ (1,16)_{k-1})$ |
| 6k+4 | $(17,\ (1,16)_{k-2},\ 1,17,3)$ |
| 6k+5 | $(17,\ (1,16)_{k-1},\ 1,4)$ |

### $F_n/F_{n-7}$

| n | QUOTIENTS |
|---|---|
| 8 | (21) |
| 9 | (34) |
| 10 | (27,2) |
| 11 | (29,1,2) |
| 12 | (28,1,4) |
| 13 | (29,8) |
| 14 | (29) |
| 15 | (29,21) |
| 16 | (29,34) |
| 17 | (29,27,2) |
| 18 | (29,29,1,2) |
| 19 | (29,28,1,4) |
| 20 | (29,29,8) |
| 21 | (29,29) |
| 22 | (29,29,21) |
| 23 | (29,29,34) |
| 24 | (29,29,27,2) |
| 25 | (29,29,29,1,2) |
| 7k | $(29_{k-1})$ |
| 7k+1 | $(29_{k-1},\ 21)$ |
| 7k+2 | $(29_{k-1}\ ,\ 34)$ |
| 7k+3 | $(29_{k-1},\ 27,2)$ |
| 7k+4 | $(29_{k},\ 1,2)$ |
| 7k+5 | $(29_{k-1},\ 28,1,4)$ |
| 7k+6 | $(29_{k},\ 8)$ |

### $F_n/F_{n-8}$

| n | QUOTIENTS |
|---|---|
| 9 | (34) |
| 10 | (55) |
| 11 | (44,2) |
| 12 | (48) |
| 13 | (46,1,1,2) |
| 14 | (47,8) |
| 15 | (46,1,12) |
| 16 | (47) |
| 17 | (46,1,33) |
| 18 | (46,1,54) |
| 19 | (46,1,43,2) |
| 20 | (46,1,47) |
| 21 | (46,1,45,1,1,2) |
| 22 | (46,1,46,8) |
| 23 | (46,1,45,1,12) |

### $F_n/F_{n-8}$

| n | QUOTIENTS |
|---|---|
| 24 | (46,1,46) |
| 25 | (46,1,45,1,33) |
| 26 | (46,1,45,1,54) |
| 27 | (46,1,45,1,43,2) |
| 28 | (46,1,45,1,47) |
| 29 | (46,1,45,1,45,1,1,2) |
| 30 | (46,1,45,1,46,8) |
| 31 | (46,1,45,1,45,1,12) |
| 32 | (46,1,45,1,46) |
| 8k | $(46,\ (1,45)_{k-3},\ 1,46)$ |
| 8k+1 | $(46,\ (1,45)_{k-2},\ 1,33)$ |
| 8k+2 | $(46,\ (1,45)_{k-2},\ 1,54)$ |
| 8k+3 | $(46,\ (1,45)_{k-2},\ 1,43,2)$ |

$L_n/L_{n-6}$

| n | QUOTIENTS |
|---|---|
| 7 | (29) |
| 8 | (15,1,2) |
| 9 | (19) |
| 10 | (17,1,1,3) |
| 11 | (18,11) |
| 12 | (17,1,8) |
| 13 | (17,1,28) |
| 14 | (17,1,14,1,2) |
| 15 | (17,1,18) |
| 16 | (17,1,16,1,1,3) |
| 17 | (17,1,17,11) |
| 18 | (17,1,16,1,8) |
| 19 | (17,1,16,1,28) |
| 20 | (17,1,16,1,14,1,2) |
| 21 | (17,1,16,1,18) |
| 22 | (17,1,16,1,16,1,1,3) |
| 23 | (17,1,16,1,17,11) |
| 24 | (17,1,16,1,16,1,8) |
| 6k | $(17, (1,16)_{k-2}, 1,8)$ |
| 6k+1 | $(17, (1,16)_{k-2}, 1,28)$ |
| 6k+2 | $(17, (1,16)_{k-2}, 1,14,1,2)$ |
| 6k+3 | $(17, (1,16)_{k-2}, 1,18)$ |
| 6k+4 | $(17, (1,16)_{k-1}, 1,1,3)$ |
| 6k+5 | $(17, (1,16)_{k-2}, 1,17,11)$ |

$L_n/L_{n-7}$

| n | QUOTIENTS |
|---|---|
| 8 | (47) |
| 9 | (25,3) |
| 10 | (30,1,3) |
| 11 | (28,2,3) |
| 12 | (29,3,1,2) |
| 13 | (28,1,17) |
| 14 | (29,14,2) |
| 15 | (29,47) |
| 16 | (29,25,3) |
| 17 | (29,30,1,3) |
| 18 | (29,28,2,3) |
| 19 | (29,29,3,1,2) |
| 20 | (29,28,1,17) |
| 21 | (29,29,14,2) |
| 22 | (29,29,47) |
| 23 | (29,29,25,3) |
| 24 | (29,29,30,1,3) |

$L_n/L_{n-7}$(cont.)

| n | QUOTIENTS |
|---|---|
| 7k | $(29_{k-1}, 14,2)$ |
| 7k+1 | $(29_{k-1}, 47)$ |
| 7k+2 | $(29_{k-1}, 25,3)$ |
| 7k+3 | $(29_{k-1}, 30,1,3)$ |
| 7k+4 | $(29_{k-1}, 28,2,3)$ |
| 7k+5 | $(29_{k}, 3,1,2)$ |
| 7k+5 | $(29_{k-1}, 28,1,17)$ |

$L_n/L_{n-8}$

| n | QUOTIENTS |
|---|---|
| 9 | (76) |
| 10 | (41) |
| 11 | (49,1,3) |
| 12 | (46) |
| 13 | (47,2,1,3) |
| 14 | (46,1,5) |
| 15 | (47,29) |
| 16 | (46,1,22,2) |
| 17 | (46,1,75) |
| 18 | (46,1,40) |
| 19 | (46,1,48,1,3) |
| 20 | (46,1,45) |
| 21 | (46,1,46,2,1,3) |
| 22 | (46,1,45,1,5) |
| 23 | (46,1,46,29) |
| 24 | (46,1,45,1,22,2) |
| 25 | (46,1,45,1,75) |
| 26 | (46,1,45,1,40) |
| 27 | (46,1,45,1,48,1,3) |
| 8k | $(46, (1,45)_{k-2}, 1,22,2)$ |
| 8k+1 | $(46, (1,45)_{k-2}, 1,75)$ |
| 8k+2 | $(46, (1,45)_{k-2}, 1,40)$ |
| 8k+3 | $(46, (1,45)_{k-2}, 1,48,1,3)$ |
| 8k+4 | $(46, (1,45)_{k-1})$ |
| 8k+5 | $(46, (1,45)_{k-2}, 1,46,2,1,3)$ |
| 8k+6 | $(46, (1,45)_{k-1}, 1,5)$ |
| 8k+7 | $(46, (1,45)_{k-2}, 1,46,29)$ |

CONTINUED FRACTION EXPANSIONS OF

QUADRATIC FIBONACCI AND LUCAS RATIOS

$F_n^2 / F_{n-1}^2$

| n | EXPANSION QUOTIENTS |
|---|---|
| 3 | (3,1) |
| 4 | (2,3,1) |
| 5 | (2,1,3,1,1) |
| 6 | (2,1,1,3,1,1,1) |
| 7 | (2,1,1,1,3,1,1,1,1) |
| 8 | (2,1,1,1,1,3,1,1,1,1,1) |
| n | $(2,\ 1_{n-4},\ 3,\ 1_{n-3})$ |

$F_n^2 / F_{n-2}^2$

| n | EXPANSION QUOTIENTS |
|---|---|
| 6 | (7,9) |
| 7 | (6,1,3,6) |
| 8 | (6,1,8,7) |
| 9 | (6,1,5,3,1,6) |
| 10 | (6,1,6,8,1,6) |
| 11 | (6,1,5,1,3,5,1,6) |
| 12 | (6,1,5,1,8,6,1,6) |
| 13 | (6,1,5,1,5,3,1,5,1,6) |
| 14 | (6,1,5,1,6,8,1,5,1,6) |
| 15 | (6,1,5,1,5,1,3,5,1,5,1,6) |
| 16 | (6,1,5,1,5,1,8,6,1,5,1,6) |
| 17 | (6,1,5,1,5,1,5,3,1,5,1,5,1,6) |
| 18 | (6,1,5,1,5,1,6,8,1,5,1,5,1,6) |
| 19 | (6,1,5,1,5,1,5,1,3,5,1,5,1,5,1,6) |
| 20 | (6,1,5,1,5,1,5,1,8,6,1,5,1,5,1,6) |
| 4k+1 | $(6,\ (1,5)_{k-1},\ 3,\ (1,5)_{k-2},\ 1,6)$ |
| 4k+2 | $(6,(1,5)k-2\ ,\ 1,6,8,\ (1,5,)_{k-2},\ 1,6)$ |
| 4k+3 | $(6,\ (1,5)_{k-1},\ 1,3,5,\ (1,5)_{k-2},\ 1,6)$ |
| 4k+4 | $(6,\ (1,5)_{k-1},\ 1,8,6,\ (1,5)_{k-2},\ 1,6)$ |

## CONTINUED FRACTION EXPANSIONS OF

## QUADRATIC FIBONACCI AND LUCAS RATIOS

### $L_n^2 / L_{n-1}^2$

| n | EXPANSION QUOTIENTS |
|---|---|
| 13 | (2,1,1,1,1,1,1,1,1,2,9,11,5,2) |
| n | (2, $1_{n-5}$, 2,9, expansion of $L_{n-3}/L_{n-8}$) |

### $L_n^2 / L_{n-2}^2$

| n | EXPANSION QUOTIENTS |
|---|---|
| 6 | (6,1,1,1,1,2,1,2) |
| 7 | (6,1,19,6) |
| 8 | (6,1,4,2,29) |
| 9 | (6,1,6,1,1,2,1,3,4) |
| 10 | (6,1,5,1,1,1,1,2,3,3,2) |
| 11 | (6,1,5,1,19,3,1,2,1,2) |
| 12 | (6,1,5,1,4,2,33,6) |
| 13 | (6,1,5,1,6,1,1,2,1,3,29) |
| 14 | (6,1,5,1,5,1,1,1,1,2,3,2,1,3,4) |
| 15 | (6,1,5,1,5,1,19,3,1,2,3,3,2) |
| 16 | (6,1,5,1,12,20,1,4,5,1,1,1,1,2) |
| 17 | (6,1,5,1,5,1,6,1,1,2,1,3,33,6) |
| 18 | (6,1,5,1,5,1,5,1,1,1,1,2,3,2,1,3,29) |
| 19 | (6,1,5,1,5,1,5,1,19,3,1,2,3,2,1,3,4) |
| 20 | (6,1,5,1,5,1,5,1,4,2,33,3,1,2,3,3,2) |
| 21 | (6,1,5,1,5,1,5,1,6,1,1,2,1,3,33,3,1,2,1,2) |
| 22 | (6,1,5,1,5,1,5,1,5,1,1,1,1,2,3,2,1,3,33,6) |
| 23 | (6,1,5,1,5,1,5,1,5,1,19,3,1,2,3,2,1,3,29) |
| 24 | (6,1,5,1,5,1,5,1,5,1,4,2,33,3,1,2,3,2,1,3,4) |
| 4k | (6, $(1,5)_{k-2}$, 1,4,2,33, expansion of $(L_{4k-6}/3)/L_{4k-11}$) |
| 4k+1 | (6, $(1,5)_{k-2}$, 1,6,1,1,2,1,3,33, expansion of $(L_{4k-10}/3)/L_{4k-15}$) |
| 4k+2 | (6, $(1,5)_{k-1}$, 1,1,1,1,2,3,2,1,3,33, expansion of $(L_{4k-14}/3)/L_{4k-19}$) |
| 4k+3 | (6, $(1,5)_{k-1}$, 1,19, expansion of $(L_{4k+2}/3)/L_{4k-3}$) |

### $L_n^2 / F_n^2$

| n | EXPANSION QUOTIENTS |
|---|---|
| 1 | (1) |
| 2 | (9) |
| 3 | (4) |
| 4 | (5,2,4) |
| 5 | (4,1,5,4) |
| 6 | (5,16) |
| 7 | (4,1,41,4) |
| 8 | (5,110,4) |
| 9 | (4,1,288) |
| 10 | (5,756,4) |

## CONTINUED FRACTION EXPANSION OF MULTIPLES OF THE GOLDEN SECTION RATIO

$$r = \frac{1 + \sqrt{5}}{2}$$

| n | Expansion of nr |
|---|---|
| 1 | $(\overline{1})$ |
| 2 | $(3,\overline{4})$ |
| 3 | $(4,\overline{1,5})$ |
| 4 | $(6,\overline{2,8})$ |
| 5 | $(8,\overline{11})$ |
| 6 | $(9,\overline{1,2,2,2,1,12})$ |
| 7 | $(11,\overline{3,15})$ |
| 8 | $(12,\overline{1,16})$ |
| 9 | $(14,\overline{1,1,3,1,1,19})$ |
| 10 | $(16,\overline{5,1,1,5,22})$ |
| 11 | $(17,\overline{1,3,1,23})$ |
| 12 | $(19,\overline{2,2,2,26})$ |
| 13 | $(21,\overline{29})$ |
| 14 | $(22,\overline{1,1,1,7,6,7,1,1,1,30})$ |
| 15 | $(24,\overline{3,1,2,3,2,1,3,33})$ |
| 16 | $(25,\overline{1,7,1,34})$ |
| 17 | $(27,\overline{1,1,37})$ |
| 18 | $(29,\overline{8,40})$ |
| 19 | $(30,\overline{1,2,1,7,1,2,1,41})$ |
| 20 | $(32,\overline{2,1,3,2,1,1,10,1,1,2,3,1,2,44})$ |
| 21 | $(33,\overline{1,45})$ |
| 22 | $(35,\overline{1,1,2,11,1,8,1,11,2,1,1,48})$ |
| 23 | $(37,\overline{4,1,1,1,9,1,1,1,4,51})$ |
| 24 | $(38,\overline{1,4,1,52})$ |
| 25 | $(40,\overline{2,4,1,1,2,2,1,1,4,2,55})$ |
| 26 | $(42,\overline{14,1,1,14,58})$ |
| 27 | $(43,\overline{1,2,5,6,1,1,11,1,1,6,5,2,1,59})$ |
| 28 | $(45,\overline{3,3,1,1,2,1,1,3,3,62})$ |
| 29 | $(46,\overline{1,11,1,63})$ |

## CONTINUED FRACTION EXPANSIONS OF MULTIPLES OF THE GOLDEN SECTION RATIO

| n | Expansion of nr |
|---|---|
| 61 | $(98,\overline{1,2,2,1,135})$ |
| 62 | $(100,\overline{3,6,1,26,1,6,3,138})$ |
| 63 | $(101,\overline{1,14,1,1,1,14,1,139})$ |
| 64 | $(103,\overline{1,1,4,8,1,2,1,1,2,35,2,1,1,2,1,8,4,1,1,142})$ |
| 65 | $(105,\overline{5,1,4,5,1,1,1,1,5,4,1,5,145})$ |
| 66 | $(106,\overline{1,3,1,3,3,2,1,36,5,16,5,36,1,2,3,3,1,3,1,146})$ |
| 67 | $(108,\overline{2,2,4,2,3,4,1,7,13,2,29,2,13,7,1,4,3,2,4,2,2,149})$ |
| 68 | $(110,\overline{38,152})$ |
| 69 | $(111,\overline{1,1,1,4,3,4,1,1,1,153})$ |
| 70 | $(113,\overline{3,1,4,3,2,1,7,1,1,5,1,2,1,2,2,4,1,38,3,5,1,13,2,1,1,2,1,1,2,13,1,5,3,38,1,4,2,2,1,2,1,5,1,1,7,1,2,3,4,1,3,156})$ |
| 71 | $(114,\overline{1,7,2,1,3,5,4,1,13,1,1,1,2,31,2,1,1,1,13,1,4,5,3,1,2,7,1,157})$ |
| 72 | $(116,\overline{2,160})$ |
| 73 | $(118,\overline{8,1,1,2,2,3,1,1,3,2,2,1,1,8,163})$ |
| 74 | $(119,\overline{1,2,1,3,3,2,40,1,14,14,1,40,2,3,3,1,2,1,164})$ |
| 75 | $(121,\overline{2,1,5,8,1,1,1,6,18,2,14,1,3,6,2,4,1,17,1,4,2,6,3,1,14,2,18,6,1,1,1,8,5,1,2,167})$ |
| 76 | $(122,\overline{1,32,1,168})$ |
| 77 | $(124,\overline{1,1,2,3,8,1,3,3,3,1,8,3,2,1,1,171})$ |
| 78 | $(126,\overline{4,1,5,4,1,2,19,43,1,1,4,2,1,18,1,2,4,1,1,43,19,2,1,4,5,1,4,174})$ |
| 79 | $(127,\overline{1,4,1,2,2,1,1,1,3,1,2,8,1,15,6,35,6,15,1,8,2,1,3,1,1,1,2,2,1,4,1,175})$ |
| 80 | $(129,\overline{2,3,1,6,2,1,1,1,5,6,1,43,1,6,5,1,1,1,2,6,1,3,2,178})$ |
| 81 | $(131,\overline{16,2,5,1,3,5,1,1,2,1,1,8,1,19,4,2,1,2,1,1,1,1,35,1,1,1,1,2,1,2,4,19,1,8,1,1,2,1,1,5,3,1,5,2,16,181})$ |
| 82 | $(132,\overline{1,2,8,1,5,45,1,2,36,2,1,45,5,1,8,2,1,182})$ |
| 83 | $(134,\overline{3,2,1,2,2,4,9,1,1,5,2,5,1,16,37,16,1,5,2,5,1,1,9,4,2,2,1,2,3,185})$ |
| 84 | $(135,\overline{1,10,1,2,1,10,1,186})$ |
| 85 | $(137,\overline{1,1,7,9,1,6,1,2,2,1,6,1,9,7,1,1,189})$ |
| 86 | $(139,\overline{6,1,1,1,2,17,9,1,1,3,1,5,2,2,1,3,1,47,3,2,9,1,2,4,38,4,2,1,9,2,3,47,1,3,1,2,2,5,1,3,1,1,9,17,2,1,1,1,6,192})$ |
| 87 | $(140,\overline{1,3,3,21,3,3,1,193})$ |
| 88 | $(142,\overline{2,1,1,2,2,9,2,2,1,1,2,196})$ |

## POWERS OF THE GOLDEN SECTION RATIO

| n | $r^{-n}$ | | |
|---|---|---|---|
| 1 | 0.6180339887 | 4989484820 | 4586834366 |
| 2 | 0.3819660112 | 5010515179 | 5413165634 |
| 3 | 0.2360679774 | 9978969640 | 9173668731 |
| 4 | 0.1458980337 | 5031545538 | 6239496903 |
| 5 | 0.0901699437 | 4947424102 | 2934171828 |
| 6 | 0.0557280900 | 0084121436 | 3305325075 |
| 7 | 0.0344418537 | 4863302665 | 9628846753 |
| 8 | 0.0212862362 | 5220818770 | 3676478322 |
| 9 | 0.0131556174 | 9642483895 | 5952368432 |
| 10 | 0.0081306187 | 5578334874 | 7724109890 |
| 11 | 0.0050249987 | 4064149020 | 8228258542 |
| 12 | 0.0031056200 | 1514185853 | 9495851348 |
| 13 | 0.0019193787 | 2549963166 | 8732407194 |
| 14 | 0.0011862412 | 8964222687 | 0763444154 |
| 15 | 0.0007331374 | 3585740479 | 7968963039 |
| 16 | 0.0004531038 | 5378482207 | 2794481115 |
| 17 | 0.0002800335 | 8207258272 | 5174481924 |
| 18 | 0.0001730702 | 7171223934 | 7619999191 |
| 19 | 0.0001069633 | 1036034337 | 7554482733 |
| 20 | 0.0000661069 | 6135189597 | 0065516458 |
| 21 | 0.0000408563 | 4900844740 | 7488966275 |
| 22 | 0.0000252506 | 1234344856 | 2576550183 |
| 23 | 0.0000156057 | 3666499884 | 4912416092 |
| 24 | 0.0000096448 | 7567844971 | 7664134092 |
| 25 | 0.0000059608 | 6098654912 | 7248282000 |
| 26 | 0.0000036840 | 1469190059 | 0415852092 |
| 27 | 0.0000022768 | 4629464853 | 6832429908 |
| 28 | 0.0000014071 | 6839725205 | 3583422184 |
| 29 | 0.0000008696 | 7789739648 | 3249007724 |
| 30 | 0.0000005374 | 9049985557 | 0334414461 |
| 31 | 0.0000003321 | 8739754091 | 2914593263 |
| 32 | 0.0000002053 | 0310231465 | 7419821197 |
| 33 | 0.0000001268 | 8429522625 | 5494772066 |
| 34 | 0.0000000784 | 1880708840 | 1925049132 |
| 35 | 0.0000000484 | 6548813785 | 3569722934 |
| 36 | 0.0000000299 | 5331895054 | 8355326198 |
| 37 | 0.0000000185 | 1216918730 | 5214396736 |
| 38 | 0.0000000114 | 4114976324 | 3140929462 |
| 39 | 0.0000000070 | 7101942406 | 2073467274 |
| 40 | 0.0000000043 | 7013033918 | 1067462187 |

# BINOMIAL COEFFICIENTS $\binom{n}{r}$

| r | n = 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 26 | 27 | 28 | 29 | 30 |
| 2 | 325 | 351 | 378 | 406 | 435 |
| 3 | 2600 | 2925 | 3276 | 3654 | 4060 |
| 4 | 14950 | 17550 | 20475 | 23751 | 27405 |
| 5 | 65780 | 80730 | 98280 | 118755 | 142506 |
| 6 | 230230 | 296010 | 376740 | 475020 | 593775 |
| 7 | 657800 | 888030 | 1184040 | 1560780 | 2035800 |
| 8 | 1562275 | 2220075 | 3108105 | 4292145 | 5852925 |
| 9 | 3124550 | 4686825 | 6906900 | 10015005 | 14307150 |
| 10 | 5311735 | 8436285 | 13123110 | 20030010 | 30045015 |
| 11 | 7726160 | 13037895 | 21474180 | 34597290 | 54627300 |
| 12 | 9657700 | 17383860 | 30421755 | 51895935 | 86493225 |
| 13 | 10400600 | 20058300 | 37442160 | 67863915 | 119759850 |
| 14 | | 20058300 | 40116600 | 77558760 | 145422675 |
| 15 | | | 37442160 | 77558760 | 155117520 |
| 16 | | | | | 145422675 |

| r | n = 31 | 32 | 33 | 34 | 35 |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 31 | 32 | 33 | 34 | 35 |
| 2 | 465 | 496 | 528 | 561 | 595 |
| 3 | 4495 | 4960 | 5456 | 5984 | 6545 |
| 4 | 31465 | 35960 | 40920 | 46376 | 52360 |
| 5 | 169911 | 201376 | 237336 | 278256 | 324632 |
| 6 | 736281 | 906192 | 1107568 | 1344904 | 1623160 |
| 7 | 2629575 | 3365856 | 4272048 | 5379616 | 6724520 |
| 8 | 7888725 | 10518300 | 13884156 | 18156204 | 23535820 |
| 9 | 20160075 | 28048800 | 38567100 | 52451256 | 70607460 |
| 10 | 44352165 | 64512240 | 92561040 | 131128140 | 183579396 |
| 11 | 84672315 | 129024480 | 193536720 | 286097760 | 417225900 |
| 12 | 141120525 | 225792840 | 354817320 | 548354040 | 834451800 |
| 13 | 206253075 | 347373600 | 573166440 | 927983760 | 1476337800 |
| 14 | 265182525 | 471435600 | 818809200 | 1391975640 | 2319959400 |
| 15 | 300540195 | 565722720 | 1037158320 | 1855967520 | 3247943160 |
| 16 | 300540195 | 601080390 | 1166803110 | 2203961430 | 4059928950 |
| 17 | | | 1166803110 | 2333606220 | 4537567650 |
| 18 | | | | | 4537567650 |

BINOMIAL COEFFICIENTS $\binom{n}{r}$

| r | n = 44 | 45 | 46 | 47 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 44 | 45 | 46 | 47 |
| 2 | 946 | 990 | 1035 | 1081 |
| 3 | 13244 | 14190 | 15180 | 16215 |
| 4 | 135751 | 148995 | 163185 | 178365 |
| 5 | 1086008 | 1221759 | 1370754 | 1533939 |
| 6 | 7059052 | 8145060 | 9366819 | 10737573 |
| 7 | 38320568 | 45379620 | 53524680 | 62891499 |
| 8 | 177232627 | 215553195 | 260932815 | 314457495 |
| 9 | 708930508 | 886163135 | 1101716330 | 1362649145 |
| 10 | 2481256778 | 3190187286 | 4076350421 | 5178066751 |
| 11 | 7669339132 | 10150595910 | 13340783196 | 17417133617 |
| 12 | 21090682613 | 28760021745 | 38910617655 | 52251400851 |
| 13 | 51915526432 | 73006209045 | 101766230790 | 104676848445 |
| 14 | 114955808528 | 166871334960 | 239877544005 | 341643774795 |
| 15 | 229911617056 | 344867425584 | 511738760544 | 751616304549 |
| 16 | 416714805914 | 646626422970 | 991493848554 | 1503232609098 |
| 17 | 686353797976 | 1103068603890 | 1749695026860 | 2741188875414 |
| 18 | 1029530696964 | 1715884494940 | 2818953098830 | 4568648125690 |
| 19 | 1408831480056 | 2438362177020 | 4154246671960 | 6973199770790 |
| 20 | 1761039350070 | 3169870830126 | 5608233007146 | 9762479679106 |
| 21 | 2012616400080 | 3773655750150 | 6943526580276 | 12551759587422 |
| 22 | 2104098963720 | 4116715363800 | 7890371113950 | 14833897694226 |
| 23 | | | 8233430727600 | 16123801841550 |

| r | n = 48 | 49 | 50 |
|---|---|---|---|
| 0 | 1 | 1 | 1 |
| 1 | 48 | 49 | 50 |
| 2 | 1128 | 1176 | 1225 |
| 3 | 17296 | 18424 | 19600 |
| 4 | 194580 | 211876 | 230300 |
| 5 | 1712304 | 1906884 | 2118760 |
| 6 | 12271512 | 13983816 | 15890700 |
| 7 | 73629072 | 85900584 | 99884400 |
| 8 | 377348994 | 450978066 | 536878650 |
| 9 | 1677106640 | 2054455634 | 2505433700 |
| 10 | 6540715896 | 8217822536 | 10272278170 |
| 11 | 22595200368 | 29135916264 | 37353738800 |
| 12 | 69668534468 | 92263734836 | 121399651100 |
| 13 | 192928249296 | 262596783764 | 354860518600 |
| 14 | 482320623240 | 675248872536 | 937845656300 |
| 15 | 1093260079344 | 1575580702584 | 2250829575120 |
| 16 | 2254848913647 | 3348108992991 | 4923689695575 |
| 17 | 4244421484512 | 6499270398159 | 9847379391150 |
| 18 | 7309837001104 | 11554258485616 | 18053528883775 |
| 19 | 11541847896480 | 18851684897584 | 30405943383200 |
| 20 | 16735679449896 | 28277527346376 | 47129212243960 |
| 21 | 22314239266528 | 39049918716424 | 67327446062800 |
| 22 | 27385657281648 | 49699896548176 | 88749815264600 |
| 23 | 30957699535776 | 58343356817424 | 108043253365600 |
| 24 | 32247603683100 | 63205303218876 | 121548660036300 |
| 25 | | | 126410606437752 |

DIAGONAL SUMS OF PASCAL'S TRIANGLE

For the values of n = 0,1,2,...,19, read the quantities horizontally by rows.

q=0, p = 1

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 |

q=0,p=2

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 2 | 2 | 3 |
| 3 | 4 | 4 | 5 | 5 |
| 6 | 6 | 7 | 7 | 8 |
| 8 | 9 | 9 | 10 | 10 |

q=0,p=3

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 2 |
| 2 | 3 | 3 | 3 | 4 |
| 4 | 4 | 5 | 5 | 5 |
| 6 | 6 | 6 | 7 | 7 |

q=0,p=4

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 2 |
| 2 | 2 | 2 | 3 | 3 |
| 3 | 3 | 4 | 4 | 4 |
| 4 | 5 | 5 | 5 | 5 |

q=0,p=5

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 |

q=1,p= 0

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 4 | 8 | 16 |
| 32 | 64 | 128 | 256 | 512 |
| 1024 | 2048 | 4096 | 8192 | 16384 |
| 32768 | 65536 | 131072 | 262144 | 524288 |

q=1,p=1

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 |
| 8 | 13 | 21 | 34 | 55 |
| 89 | 144 | 233 | 377 | 610 |
| 987 | 1597 | 2584 | 4181 | 6765 |

q=1,p=2

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 3 |
| 4 | 6 | 9 | 13 | 19 |
| 28 | 41 | 60 | 88 | 129 |
| 189 | 277 | 406 | 595 | 872 |

DIAGONAL SUMS OF PASCAL'S TRIANGLE

q=3,p=-2

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 6 | 19 | 60 |
| 189 | 595 | 1873 | 5896 | 18560 |
| 58425 | 183916 | 578949 | 1822473 | 5736961 |
| 18059374 | 56849086 | 178955183 | 563332848 | 1773314929 |

q=3,p=-1

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 2 | 5 | 12 |
| 28 | 65 | 151 | 351 | 816 |
| 1897 | 4410 | 10252 | 23833 | 55405 |
| 128801 | 299426 | 696081 | 1618192 | 3761840 |

q=3,p=0

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 5 |
| 11 | 22 | 43 | 85 | 170 |
| 341 | 683 | 1366 | 2731 | 5461 |
| 10922 | 21845 | 43691 | 87382 | 174763 |

q=3,p=1

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 2 |
| 5 | 11 | 21 | 37 | 64 |
| 113 | 205 | 377 | 693 | 1266 |
| 2301 | 4175 | 7581 | 13785 | 25088 |

q=3,p=2

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 5 | 11 | 21 | 36 |
| 58 | 92 | 149 | 250 | 431 |
| 750 | 1299 | 2227 | 3784 | 6401 |

q=3,p=3

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 5 | 11 | 21 |
| 36 | 57 | 86 | 128 | 194 |
| 305 | 497 | 827 | 1381 | 2287 |

q=4,p=-3

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 7 | 26 | 95 |
| 345 | 1252 | 4544 | 16493 | 59864 |
| 217286 | 788674 | 2862617 | 10390321 | 37713313 |
| 136886433 | 496850954 | 1803399103 | 6545722210 | 23758733815 |

q=4,p=-2

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 2 | 6 | 17 |
| 45 | 117 | 305 | 798 | 2090 |
| 5473 | 14329 | 37513 | 98209 | 257114 |
| 673134 | 1762289 | 4613733 | 12078909 | 31622993 |

q=4,p=-1

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 6 |
| 16 | 37 | 80 | 172 | 377 |
| 839 | 1874 | 4175 | 9274 | 20577 |
| 45665 | 101393 | 225193 | 500162 | 1110790 |

# DIAGONAL SUMS OF PASCAL'S TRIANGLE

q=6,p=-3

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 8 |
| 29 | 86 | 224 | 554 | 1381 |
| 3556 | 9382 | 24901 | 65737 | 172321 |
| 450017 | 1174985 | 3072365 | 8044478 | 21074012 |

q=6, p=-2

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 2 |
| 8 | 29 | 85 | 212 | 476 |
| 1016 | 2172 | 4825 | 11213 | 26763 |
| 64095 | 151851 | 354737 | 820328 | 1889968 |

q=6, p=-1

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 8 | 29 | 85 | 211 |
| 464 | 938 | 1808 | 3459 | 6826 |
| 14198 | 30960 | 69143 | 154433 | 340006 |

q=6, p=0

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 8 | 29 | 85 |
| 211 | 463 | 926 | 1730 | 3095 |
| 5461 | 9829 | 18565 | 37130 | 77540 |

q=6, p=1

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 2 | 8 | 29 |
| 85 | 211 | 463 | 925 | 1718 |
| 3017 | 5097 | 8464 | 14197 | 24753 |

q=7, p=-4

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 9 |
| 37 | 122 | 346 | 913 | 2398 |
| 6515 | 18317 | 52226 | 148408 | 417810 |
| 1168085 | 3258813 | 9103828 | 25488736 | 71462437 |

q=7, p=-3

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 2 |
| 9 | 37 | 121 | 332 | 808 |
| 1837 | 4113 | 9497 | 23091 | 58462 |
| 150129 | 382810 | 960520 | 2373982 | 5816480 |

q=7,p=-2

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 9 | 37 | 121 | 331 |
| 794 | 1732 | 3553 | 7116 | 14501 |
| 31078 | 70607 | 166922 | 399315 | 946121 |

SPECIAL DIAGONAL SUMS OF PASCAL'S TRIANGLE

R = 2

| n | H(n) | n | H(n) |
|---|---|---|---|
| 1 | 1 | 41 | 267914296 |
| 2 | 2 | 42 | 433494437 |
| 3 | 3 | 43 | 701408733 |
| 4 | 5 | 44 | 1134903170 |
| 5 | 8 | 45 | 1836311903 |
| 6 | 13 | 46 | 2971215073 |
| 7 | 21 | 47 | 4807526976 |
| 8 | 34 | 48 | 7778742049 |
| 9 | 55 | 49 | 1258626902 5 |
| 10 | 89 | 50 | 2036501107 4 |
| 11 | 144 | 51 | 3295128009 9 |
| 12 | 233 | 52 | 5331629117 3 |
| 13 | 377 | 53 | 8626757127 2 |
| 14 | 610 | 54 | 1395838624 45 |
| 15 | 987 | 55 | 2258514337 17 |
| 16 | 1597 | 56 | 3654352961 62 |
| 17 | 2584 | 57 | 5912867298 79 |
| 18 | 4181 | 58 | 9567220260 41 |
| 19 | 6765 | 59 | 1548008755 920 |
| 20 | 10946 | 60 | 2504730781 961 |
| 21 | 17711 | 61 | 4052739537 881 |
| 22 | 28657 | 62 | 6557470319 842 |
| 23 | 46368 | 63 | 1061020985 7723 |
| 24 | 75025 | 64 | 1716768017 7565 |
| 25 | 121393 | 65 | 2777789003 5288 |
| 26 | 196418 | 66 | 4494557021 2853 |
| 27 | 317811 | 67 | 7272346024 8141 |
| 28 | 514229 | 68 | 1176690304 60994 |
| 29 | 832040 | 69 | 1903924907 09135 |
| 30 | 1346269 | 70 | 3080615211 70129 |
| 31 | 2178309 | 71 | 4984540118 79264 |
| 32 | 3524578 | 72 | 8065155330 49393 |
| 33 | 5702887 | 73 | 1304969544 928657 |
| 34 | 9227465 | 74 | 2111485077 978050 |
| 35 | 14930352 | 75 | 3416454622 906707 |
| 36 | 24157817 | 76 | 5527939700 884757 |
| 37 | 39088169 | 77 | 8944394323 791464 |
| 38 | 63245986 | 78 | 1447233402 4676221 |
| 39 | 102334155 | 79 | 2341672834 8467685 |
| 40 | 165580141 | 80 | 3788906237 3143906 |

SPECIAL DIAGONAL SUMS OF PASCAL'S TRIANGLE

R = 4

| n | H(n) | n | H(n) |
|---|---|---|---|
| 1 | 1 | 41 | 571388 |
| 2 | 2 | 42 | 788674 |
| 3 | 3 | 43 | 1088589 |
| 4 | 4 | 44 | 1502555 |
| 5 | 5 | 45 | 2073943 |
| 6 | 7 | 46 | 2862617 |
| 7 | 10 | 47 | 3951206 |
| 8 | 14 | 48 | 5453761 |
| 9 | 19 | 49 | 7527704 |
| 10 | 26 | 50 | 10390321 |
| 11 | 36 | 51 | 14341527 |
| 12 | 50 | 52 | 19795288 |
| 13 | 69 | 53 | 27322992 |
| 14 | 95 | 54 | 37713313 |
| 15 | 131 | 55 | 52054840 |
| 16 | 181 | 56 | 71850128 |
| 17 | 250 | 57 | 99173120 |
| 18 | 345 | 58 | 136886433 |
| 19 | 476 | 59 | 188941273 |
| 20 | 657 | 60 | 260791401 |
| 21 | 907 | 61 | 359964521 |
| 22 | 1252 | 62 | 496850954 |
| 23 | 1728 | 63 | 685792227 |
| 24 | 2385 | 64 | 946583628 |
| 25 | 3292 | 65 | 1306548149 |
| 26 | 4544 | 66 | 1803399103 |
| 27 | 6272 | 67 | 2489191330 |
| 28 | 8657 | 68 | 3435774958 |
| 29 | 11949 | 69 | 4742323107 |
| 30 | 16493 | 70 | 6545722210 |
| 31 | 22765 | 71 | 9034913540 |
| 32 | 31422 | 72 | 1247068849 8 |
| 33 | 43371 | 73 | 1721301160 5 |
| 34 | 59864 | 74 | 2375873381 5 |
| 35 | 82629 | 75 | 3279364735 5 |
| 36 | 114051 | 76 | 4526433585 3 |
| 37 | 157422 | 77 | 6247734745 8 |
| 38 | 217286 | 78 | 8623608127 3 |
| 39 | 299915 | 79 | 1190297286 28 |
| 40 | 413966 | 80 | 1642940644 81 |

SPECIAL DIAGONAL SUMS OF PASCAL'S TRIANGLE

R=6

| n | H(n) |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |
| 7 | 7 |
| 8 | 9 |
| 9 | 12 |
| 10 | 16 |
| 11 | 21 |
| 12 | 27 |
| 13 | 34 |
| 14 | 43 |
| 15 | 55 |
| 16 | 71 |
| 17 | 92 |
| 18 | 119 |
| 19 | 153 |
| 20 | 196 |
| 21 | 251 |
| 22 | 322 |
| 23 | 414 |
| 24 | 533 |
| 25 | 686 |
| 26 | 882 |
| 27 | 1133 |
| 28 | 1455 |
| 29 | 1869 |
| 30 | 2402 |
| 31 | 3088 |
| 32 | 3970 |
| 33 | 5103 |
| 34 | 6558 |
| 35 | 8427 |
| 36 | 10829 |
| 37 | 13917 |
| 38 | 17887 |
| 39 | 22990 |
| 40 | 29548 |
| 41 | 37975 |
| 42 | 48804 |
| 43 | 62721 |
| 44 | 809083 |
| 45 | 103598 |
| 46 | 133146 |
| 47 | 171121 |
| 48 | 219925 |
| 49 | 282646 |
| 50 | 363254 |
| 51 | 466852 |
| 52 | 599998 |
| 53 | 771119 |
| 54 | 991044 |
| 55 | 1273690 |
| 56 | 1636944 |
| 57 | 2103796 |
| 58 | 2703794 |
| 59 | 3474913 |
| 60 | 4465957 |
| 61 | 5739647 |
| 62 | 7376591 |
| 63 | 9480387 |
| 64 | 12184181 |
| 65 | 15659094 |
| 66 | 20125051 |
| 67 | 25864698 |
| 68 | 33241289 |
| 69 | 42721676 |
| 70 | 54905857 |
| 71 | 70564951 |
| 72 | 90690002 |
| 73 | 116554700 |
| 74 | 149795989 |
| 75 | 192517665 |
| 76 | 247423522 |
| 77 | 317988473 |
| 78 | 408678475 |
| 79 | 525233175 |
| 80 | 675029164 |

SPECIAL DIAGONAL SUMS OF PASCAL'S TRIANGLE

R= 8

| n | H(n) | n | H(n) |
|---|---|---|---|
| 1 | 1 | 41 | 7837 |
| 2 | 2 | 42 | 9661 |
| 3 | 3 | 43 | 11908 |
| 4 | 4 | 44 | 14674 |
| 5 | 5 | 45 | 18078 |
| 6 | 6 | 46 | 22268 |
| 7 | 7 | 47 | 27428 |
| 8 | 8 | 48 | 33786 |
| 9 | 9 | 49 | 41623 |
| 10 | 11 | 50 | 51284 |
| 11 | 14 | 51 | 63192 |
| 12 | 18 | 52 | 77866 |
| 13 | 23 | 53 | 95944 |
| 14 | 29 | 54 | 118212 |
| 15 | 36 | 55 | 145640 |
| 16 | 44 | 56 | 179426 |
| 17 | 53 | 57 | 221049 |
| 18 | 64 | 58 | 272333 |
| 19 | 78 | 59 | 335525 |
| 20 | 96 | 60 | 413391 |
| 21 | 119 | 61 | 509335 |
| 22 | 148 | 62 | 627547 |
| 23 | 184 | 63 | 773187 |
| 24 | 228 | 64 | 952613 |
| 25 | 281 | 65 | 1173662 |
| 26 | 345 | 66 | 1445995 |
| 27 | 423 | 67 | 1781520 |
| 28 | 519 | 68 | 2194911 |
| 29 | 638 | 69 | 2704246 |
| 30 | 786 | 70 | 3331793 |
| 31 | 970 | 71 | 4104980 |
| 32 | 1198 | 72 | 5057593 |
| 33 | 1479 | 73 | 6231255 |
| 34 | 1824 | 74 | 7677250 |
| 35 | 2247 | 75 | 9458770 |
| 36 | 2766 | 76 | 11653681 |
| 37 | 3404 | 77 | 14357927 |
| 38 | 4190 | 78 | 17689720 |
| 39 | 5160 | 79 | 21794700 |
| 40 | 6358 | 80 | 26852293 |

SPECIAL DIAGONAL SUMS OF PASCAL'S TRIANGLE

R = 10

| n | H(n) | n | H(n) |
|---|---|---|---|
| 1 | 1 | 41 | 2761 |
| 2 | 2 | 42 | 3311 |
| 3 | 3 | 43 | 3966 |
| 4 | 4 | 44 | 4746 |
| 5 | 5 | 45 | 5676 |
| 6 | 6 | 46 | 6787 |
| 7 | 7 | 47 | 8117 |
| 8 | 8 | 48 | 9712 |
| 9 | 9 | 49 | 11627 |
| 10 | 10 | 50 | 13927 |
| 11 | 11 | 51 | 16688 |
| 12 | 13 | 52 | 19999 |
| 13 | 16 | 53 | 23965 |
| 14 | 20 | 54 | 28711 |
| 15 | 25 | 55 | 34387 |
| 16 | 31 | 56 | 41174 |
| 17 | 38 | 57 | 49291 |
| 18 | 46 | 58 | 59003 |
| 19 | 55 | 59 | 70630 |
| 20 | 65 | 60 | 84557 |
| 21 | 76 | 61 | 101245 |
| 22 | 89 | 62 | 121244 |
| 23 | 105 | 63 | 145209 |
| 24 | 125 | 64 | 173920 |
| 25 | 150 | 65 | 208307 |
| 26 | 181 | 66 | 249481 |
| 27 | 219 | 67 | 298772 |
| 28 | 265 | 68 | 357775 |
| 29 | 320 | 69 | 428405 |
| 30 | 385 | 70 | 512962 |
| 31 | 461 | 71 | 614207 |
| 32 | 550 | 72 | 735451 |
| 33 | 655 | 73 | 880660 |
| 34 | 780 | 74 | 1054580 |
| 35 | 930 | 75 | 1262887 |
| 36 | 1111 | 76 | 1512368 |
| 37 | 1330 | 77 | 1811140 |
| 38 | 1595 | 78 | 2168915 |
| 39 | 1915 | 79 | 2597320 |
| 40 | 2300 | 80 | 3110282 |

SPECIAL DIAGONAL SUMS OF PASCAL'S TRIANGLE

R = 12

| n | H(n) | n | H(n) |
|---|---|---|---|
| 1 | 1 | 41 | 1298 |
| 2 | 2 | 42 | 1516 |
| 3 | 3 | 43 | 1774 |
| 4 | 4 | 44 | 2080 |
| 5 | 5 | 45 | 2443 |
| 6 | 6 | 46 | 2873 |
| 7 | 7 | 47 | 3381 |
| 8 | 8 | 48 | 3979 |
| 9 | 9 | 49 | 4680 |
| 10 | 10 | 50 | 5499 |
| 11 | 11 | 51 | 6454 |
| 12 | 12 | 52 | 7567 |
| 13 | 13 | 53 | 8865 |
| 14 | 15 | 54 | 10381 |
| 15 | 18 | 55 | 12155 |
| 16 | 22 | 56 | 14235 |
| 17 | 27 | 57 | 16678 |
| 18 | 33 | 58 | 19551 |
| 19 | 40 | 59 | 22932 |
| 20 | 48 | 60 | 26911 |
| 21 | 57 | 61 | 31591 |
| 22 | 67 | 62 | 37090 |
| 23 | 78 | 63 | 43544 |
| 24 | 90 | 64 | 51111 |
| 25 | 103 | 65 | 59976 |
| 26 | 118 | 66 | 70357 |
| 27 | 136 | 67 | 82512 |
| 28 | 158 | 68 | 96747 |
| 29 | 185 | 69 | 113425 |
| 30 | 218 | 70 | 132976 |
| 31 | 258 | 71 | 155908 |
| 32 | 306 | 72 | 182819 |
| 33 | 363 | 73 | 214410 |
| 34 | 430 | 74 | 251500 |
| 35 | 508 | 75 | 295044 |
| 36 | 598 | 76 | 346155 |
| 37 | 701 | 77 | 406131 |
| 38 | 819 | 78 | 476488 |
| 39 | 955 | 79 | 559000 |
| 40 | 1113 | 80 | 655747 |

## TERMS OF THE SEQUENCE WITH $T_0 = 0, T_1 = 1$ AND $T_{n+1} = 3T_n + T_{n-1}$

| n | $T_n$ |
|---|---|
| 41 | 5212790777 1794512006 9 |
| 42 | 1721667838 3462321945 20 |
| 43 | 5686282592 7566417036 29 |
| 44 | 1878051561 6616157305 407 |
| 45 | 6202782944 2605113619 850 |
| 46 | 2048640039 4443149816 4957 |
| 47 | 6766198412 7589960811 4721 |
| 48 | 2234723527 7721303225 09120 |
| 49 | 7380790424 5922905756 42081 |
| 50 | 2437709480 1549002049 435363 |
| 51 | 8051207482 9239296723 948170 |
| 52 | 2659133192 8926689222 1279873 |
| 53 | 8782520326 9703997338 7787789 |
| 54 | 2900669417 3803868123 84643240 |
| 55 | 9580260284 8382004105 41717509 |
| 56 | 3164145027 1894988044 009795767 |
| 57 | 1045046111 0052316454 2571104810 |
| 58 | 3451552835 7346448167 1723110197 |
| 59 | 1139970461 8209166095 5774043540 1 |
| 60 | 3765066669 0362143103 4494441640 0 |
| 61 | 1243517046 8929559540 5925736846 01 |
| 62 | 4107057807 5824892932 1226654702 03 |
| 63 | 1356469046 9640423833 6960570095 210 |
| 64 | 4480112921 6503760794 3004375755 833 |
| 65 | 1479680781 1915170621 6597369736 2709 |
| 66 | 4887053635 7395887944 4092546784 3960 |
| 67 | 1614084168 8410283445 4887501008 94589 |
| 68 | 5330957870 0970439130 9071757705 27727 |
| 69 | 1760695777 9132160083 8210277412 477770 |
| 70 | 5815183120 7493524164 5538008007 961037 |
| 71 | 1920624514 0161273257 7482430143 6360881 |
| 72 | 6343391854 1233172189 7001091231 7043680 |
| 73 | 2095080007 6386078982 6848570383 87491921 |
| 74 | 6919579208 3281554167 0245820274 79519443 |
| 75 | 2285381763 2623074148 3758603120 826050250 |
| 76 | 7548103210 6197377861 8300391389 957670193 |
| 77 | 2492969139 5121520773 3865977729 0699060829 |
| 78 | 8233717739 5984300106 3427972326 2054852680 |
| 79 | 2719412235 8307442109 2414989470 7686361886 9 |
| 80 | 8981608481 4520756338 3587765644 9264570928 7 |

## TERMS OF THE SEQUENCE WITH $T_0=2$, $T_1 = 3$

## AND $T_{n+1} = 3T_n + T_{n-1}$

| n | $T_n$ |
|---|---|
| 1 | 3 |
| 2 | 11 |
| 3 | 36 |
| 4 | 119 |
| 5 | 393 |
| 6 | 1298 |
| 7 | 4287 |
| 8 | 14159 |
| 9 | 46764 |
| 10 | 154451 |
| 11 | 510117 |
| 12 | 1684802 |
| 13 | 5564523 |
| 14 | 18378371 |
| 15 | 60699636 |
| 16 | 200477279 |
| 17 | 662131473 |
| 18 | 2186871698 |
| 19 | 7222746567 |
| 20 | 2385511139 9 |
| 21 | 7878808076 4 |
| 22 | 2602193536 91 |
| 23 | 8594461418 37 |
| 24 | 2838557779 202 |
| 25 | 9375119479 443 |
| 26 | 3096391621 7531 |
| 27 | 1022668681 32036 |
| 28 | 3377645206 13639 |
| 29 | 1115560429 972953 |
| 30 | 3684445810 532498 |
| 31 | 1216889786 1570447 |
| 32 | 4019113939 5243839 |
| 33 | 1327423160 47301964 |
| 34 | 4384180875 37149731 |
| 35 | 1447996578 658751157 |
| 36 | 4782407823 513403202 |
| 37 | 1579522004 9198960763 |
| 38 | 5216806797 1110285491 |
| 39 | 1722994239 6252981723 6 |
| 40 | 5690663398 5869973719 9 |

TERMS OF THE SEQUENCE WITH $T_0=2$, $T_1=3$

AND $T_{n+1} = 3T_n + T_{n-1}$

| n | $T_n$ |
|---|---|
| 81 | 1069559300 0346506460 4967103905 0649693658 764 |
| 82 | 3532514399 2641290913 8257244473 9580745341 891 |
| 83 | 1166710249 7827037920 1973883732 6939192968 4437 |
| 84 | 3853382189 2745242851 9747375645 4775653439 5202 |
| 85 | 1272685681 7606276647 6121601066 9126615328 70043 |
| 86 | 4203395264 2093354228 0339540765 2857411330 05331 |
| 87 | 1388287147 4388633933 1714022336 2769884931 886036 |
| 88 | 4585200968 7375237222 3176021085 3595395928 663439 |
| 89 | 1514389005 3651434560 0124208559 2355607271 7876353 |
| 90 | 5001687112 9691827402 2690227786 2426361408 2292498 |
| 91 | 1651945034 4272691676 6819489191 7963469149 64753847 |
| 92 | 5456003814 5787257770 2727490354 0133043589 76554039 |
| 93 | 1801995647 8163446498 7500196025 3836259991 894415964 |
| 94 | 5951587324 9069065273 2773337111 5522084334 659801931 |
| 95 | 1965675762 2537064231 8582020736 0040251299 5873821757 |
| 96 | 6492186019 2518099222 9023395919 1672962332 2281267202 |
| 97 | 2144223382 0009136190 0565220849 3505913829 6271762336 3 |
| 98 | 7081888747 9279218492 4598002139 9685037722 1043413729 1 |
| 99 | 2338988962 5784679166 7435922726 9256102699 5940200352 36 |
| 100 | 7725155762 5281959349 4767568394 7736811870 9924942429 99 |

## ENTRY POINTS IN THE (3,1) SEQUENCE

| p | E(p) | p | E(p) | p | E(p) | p | E(p) |
|---|---|---|---|---|---|---|---|
| 811 | 812 | 1109 | 277 | 1453 | 121 | 1783 | 1784 |
| 821 | 137 | 1117 | 279 | 1459 | 1458 | 1787 | 1788 |
| 823 | 822 | 1123 | 1124 | 1471 | 1472 | 1789 | 895 |
| 827 | 92 | 1129 | 565 | 1481 | 740 | 1801 | 901 |
| 829 | 414 | 1151 | 384 | 1483 | 1482 | 1811 | 1810 |
| 839 | 840 | 1153 | 576 | 1487 | 496 | 1823 | 1822 |
| 853 | 427 | 1163 | 1164 | 1489 | 149 | 1831 | 1832 |
| 857 | 107 | 1171 | 1170 | 1493 | 747 | 1847 | 1846 |
| 859 | 286 | 1181 | 591 | 1499 | 1498 | 1861 | 49 |
| 863 | 864 | 1187 | 1186 | 1511 | 1510 | 1867 | 1868 |
| 877 | 439 | 1193 | 298 | 1523 | 1524 | 1871 | 374 |
| 881 | 110 | 1201 | 601 | 1531 | 34 | 1873 | 234 |
| 883 | 98 | 1213 | 303 | 1543 | 514 | 1877 | 939 |
| 887 | 886 | 1217 | 87 | 1549 | 775 | 1879 | 1880 |
| 907 | 906 | 1223 | 1222 | 1553 | 259 | 1889 | 472 |
| 911 | 910 | 1229 | 123 | 1559 | 1558 | 1901 | 190 |
| 919 | 54 | 1231 | 1230 | 1567 | 32 | 1907 | 1906 |
| 929 | 155 | 1237 | 619 | 1571 | 1572 | 1913 | 319 |
| 937 | 117 | 1249 | 624 | 1579 | 1580 | 1931 | 644 |
| 941 | 157 | 1259 | 1260 | 1583 | 1582 | 1933 | 161 |
| 947 | 948 | 1277 | 638 | 1597 | 799 | 1949 | 487 |
| 953 | 119 | 1279 | 1280 | 1601 | 89 | 1951 | 650 |
| 967 | 968 | 1283 | 1282 | 1607 | 536 | 1973 | 493 |
| 971 | 970 | 1289 | 645 | 1609 | 268 | 1979 | 1978 |
| 977 | 489 | 1291 | 1290 | 1613 | 403 | 1987 | 1988 |
| 983 | 328 | 1297 | 9 | 1619 | 1620 | 1993 | 498 |
| 991 | 990 | 1301 | 650 | 1621 | 405 | 1997 | 999 |
| 997 | 498 | 1303 | 1302 | 1627 | 1628 | 1999 | 1998 |
| 1009 | 101 | 1307 | 1308 | 1637 | 409 | 2003 | 2002 |
| 1013 | 506 | 1319 | 1320 | 1657 | 829 | 2011 | 402 |
| 1019 | 1020 | 1321 | 661 | 1663 | 1662 | 2017 | 1009 |
| 1021 | 511 | 1327 | 442 | 1667 | 1666 | 2027 | 2026 |
| 1031 | 1030 | 1361 | 68 | 1669 | 835 | 2029 | 338 |
| 1033 | 517 | 1367 | 456 | 1693 | 94 | 2039 | 680 |
| 1039 | 1038 | 1373 | 687 | 1697 | 849 | 2053 | 171 |
| 1049 | 131 | 1381 | 690 | 1699 | 1698 | 2063 | 2062 |
| 1051 | 1052 | 1399 | 1400 | 1709 | 95 | 2069 | 115 |
| 1061 | 531 | 1409 | 235 | 1721 | 861 | 2081 | 1040 |
| 1063 | 1062 | 1423 | 1424 | 1723 | 1724 | 2083 | 2082 |
| 1069 | 89 | 1427 | 1426 | 1733 | 866 | 2087 | 2088 |
| 1087 | 1088 | 1429 | 14 | 1741 | 290 | 2089 | 1044 |
| 1091 | 1090 | 1433 | 179 | 1747 | 1748 | 2099 | 700 |
| 1093 | 546 | 1439 | 1438 | 1753 | 877 | 2111 | 2112 |
| 1097 | 549 | 1447 | 1446 | 1759 | 1758 | 2113 | 1057 |
| 1103 | 1104 | 1451 | 1452 | 1777 | 888 | 2129 | 1064 |

## FACTOR FORMS OF PRIMES IN THE (3,1) SEQUENCES

The sequences in question are defined by initial values

$Y_0 = 0,\ Y_1 = 1$ and $Z_0 = 2,\ Z_1 = 3$

with the common recursion relation $T_{n+1} = 3T_n + T_{n-1}$. In terms of the roots $r = \frac{1+\sqrt{13}}{2}$ and $s = \frac{1-\sqrt{13}}{2}$ of $x^2 - 3x - 1 = 0$,

$$Y_n = \frac{r^n - s^n}{r - s} \qquad \text{and } Z_n = r^n + s^n$$

Since $Y_{2n} = Y_n Z_n$ it is not necessary to specify the factor forms for even subscripts of the Y sequence.

### FORMS OF FACTORS OF $Y_n$ (n odd)

| n(mod 26) | FACTOR FORMS(mod 52n) |
|---|---|
| 1 | 1, 8n+1, 16n+1, 24n+1, 28n+1,48n+1<br>6n-1, 22n-1, 34n-1, 38n-1, 42n-1, 46n-1 |
| 3 | 1, 8n+1, 16n+1, 20n+1, 40n+1, 44n+1<br>2n-1, 14n-1, 30n-1, 42n-1, 46n-1, 50n-1 |
| 5 | 1, 12n+1, 16n+1, 20n+1, 24n+1, 36n+1<br>18n-1, 22n-1, 30n-1, 38n-1, 46n-1, 50n-1 |
| 7 | 1, 4n+1, 16n+1, 32n+1, 44n+1, 48n+1<br>6n-1, 14n-1, 18n-1, 38n-1, 42n-1, 50n-1 |
| 9 | 1, 20n+1, 24n+1, 32n+1, 40n+1, 48n+1<br>10n-1, 14n-1, 18n-1, 22n-1, 34n-1, 50n-1 |
| 11 | 1, 12n+1, 28n+1, 40n+1, 44n+1, 48n+1<br>2n-1, 10n-1, 18n-1, 22n-1, 42n-1, 46n-1 |
| 13 | 1(mod 4n) and 2n-1 (mod 4n) |
| 15 | 1, 4n+1, 8n+1, 12n+1, 24n+1, 40n+1<br>6n-1, 10n-1, 30n-1, 34n-1, 42n-1, 50n-1 |
| 17 | 1, 4n+1, 12n+1, 20n+1, 28n+1, 32n+1<br>2n-1,18n-1, 30n-1, 34n-1, 38n-1, 42n-1 |
| 19 | 1, 4n+1, 8n+1, 20n+1, 36n+1, 48n+1<br>2n-1, 10n-1, 14n-1, 34n-1, 38n-1, 46n-1 |
| 21 | 1, 16n+1, 28n+1, 32n+1, 36n+1, 40n+1<br>2n-1, 6n-1, 14n-1, 22n-1, 30n-1, 34n-1 |
| 23 | 1, 8n+1, 12n+1, 32n+1, 36n+1, 44n+1<br>2n-1, 6n-1, 10n-1, 22n-1, 38n-1, 50n-1 |
| 25 | 1, 4n+1, 24n+1, 28n+1, 36n+1, 44n+1<br>6n-1, 10n-1, 14n-1, 18n-1, 30n-1, 46n-1 |

# FACTOR FORMS OF PRIMES IN THE (3,1) SEQUENCES

## FORMS OF FACTORS OF $Z_n$ (n even)

| n (mod 52) | FACTOR FORMS |
|---|---|
| 12 | 1, 2n+1, 4n+1, 10n+1, 18n+1, 24n+1 (mod 26n)<br>4n-1, 6n-1, 10n-1, 14n-1, 18n-1, 20n-1 (mod 26n) |
| 14 | 1, 8n+1, 16n+1, 24n+1, 28n+1, 48n+1 (mod 52n)<br>6n-1, 22n-1, 34n-1, 38n-1, 42n-1, 46n-1 (mod 52n) |
| 16 | 1, 8n+1, 14n+1, 16n+1, 18n+1, 20n+1 (mod 26n)<br>2n-1, 4n-1, 14n-1, 16n-1, 20n-1, 24n-1 (mod 26n) |
| 18 | 1, 12n+1, 16n+1, 20n+1, 24n+1, 36n+1 (mod 52n)<br>18n-1, 22n-1, 30n-1, 38n-1, 46n-1, 50n-1 (mod 52n) |
| 20 | 1, 4n+1, 6n+1, 16n+1, 18n+1, 22n+1 (mod 26n)<br>6n-1, 12n-1, 14n-1, 16n-1, 18n-1, 24n-1 (mod 26n) |
| 22 | 1, 20n+1, 24n+1, 32n+1, 40n+1, 48n+1 (mod 52n)<br>12n-1, 16n-1, 20n-1, 24n-1, 36n-1, 52n-1 (mod 52n) |
| 24 | 1, 2n+1, 12n+1, 14n+1, 18n+1, 22n+1 (mod 26n)<br>2n-1, 10n-1, 16n-1, 18n-1, 20n-1, 22n-1 (mod 26n) |
| 26 | 1 (mod 4n) |
| 28 | 1, 4n+1, 8n+1, 12n+1, 14n+1, 24n+1 (mod 26n)<br>4n-1, 6n-1, 8n-1, 10n-1, 16n-1, 24n-1 (mod 26n) |
| 30 | 1, 4n+1, 12n+1, 20n+1, 28n+1, 32n+1 (mod 52n)<br>2n-1, 18n-1, 30n-1, 34n-1, 38n-1, 42n-1 (mod 52n) |
| 32 | 1, 4n+1, 8n+1, 10n+1, 20n+1, 22n+1 (mod 26n)<br>2n-1, 8n-1, 10n-1, 12n-1, 14n-1, 20n-1 (mod 26n) |
| 34 | 1, 16n+1, 28n+1, 32n+1, 36n+1, 40n+1 (mod 52n)<br>2n-1, 6n-1, 14n-1, 22n-1, 30n-1, 34n-1 (mod 52n) |
| 36 | 1, 6n+1, 8n+1, 10n+1, 12n+1, 18n+1 (mod 26n)<br>2n-1, 6n-1, 10n-1, 12n-1, 22n-1, 24n-1 (mod 26n) |
| 38 | 1, 4n+1, 24n+1, 28n+1, 36n+1, 44n+1 (mod 52n)<br>6n-1, 10n-1, 14n-1, 18n-1, 30n-1, 46n-1 (mod 52n) |
| 40 | 1,2n+1,8n+1,16n+1,22n+1,24n+1 (mod 26n)<br>6n-1, 8n-1, 12n-1, 16n-1, 20n-1, 22n-1 (mod 26n) |
| 42 | 1, 8n+1, 16n+1, 20n+1, 40n+1, 44n+1 (mod 52n)<br>2n-1, 14n-1, 30n-1, 42n-1, 46n-1, 50n-1 (mod 52n) |

## CHARACTERISTIC NUMBERS OF (3,1) SEQUENCES

The (3,1) sequences apart from the sequence beginning 0,1 have a smallest value for which

$$3T_n < T_{n+1}$$

This smallest value is taken as $T_1$ in identifying the sequence.

The sequences come in conjugate pairs which are placed adjacent to each other in the table. The characteristic number D of the sequence is the absolute value of

$$T_{n+1}\,T_{n-1} - T_n^2$$

| D | FACTORIZATION | SEQUENCES |
|---|---|---|
| 1 | 1 | (1,3) |
| 3 | 3 | (1,4) (2,7) |
| 9 | 3*3 | (1,5) (5,17) |
| 13 | 13 | (3,11) |
| 17 | 17 | (1,6) (8,27) |
| 23 | 23 | (2,9) (7,24) |
| 27 | 3*3*3 | (1,7) (11,37) |
| 29 | 29 | (4,15) (5,18) |
| 39 | 3*13 | (1,8) (14,47) |
| 43 | 43 | (3,13) (9,31) |
| 51 | 3*17 | (2,11) (13,44) (5,19) (7,25) |
| 53 | 53 | (1,9) (17,57) |
| 61 | 61 | (3,14) (12,41) |
| 69 | 3*23 | (1,10) (20,67) (4,17) (11,38) |
| 79 | 79 | (6,23) (9,32) |
| 81 | $3^4$ | (7,26) (8,29) |
| 87 | 3*29 | (1,11) (23,77) (2,13) (19,64) |
| 101 | 101 | (5,21) (13,45) |
| 103 | 103 | (3,16) (18,61) |
| 107 | 107 | (1,12) (26,87) |
| 113 | 113 | (7,27) (11,39) |
| 117 | 3*3*13 | (4,19) (17,58) |
| 127 | 127 | (3,17) (21,71) |
| 129 | 3*43 | (1,13) (29,97) (5,22) (16,55) |
| 131 | 131 | (2,15) (25,84) |
| 139 | 139 | (6,25) (15,52) |
| 153 | 3*3*17 | (1,14) (32,107) (8,31) (13,46) |
| 157 | 157 | (9,34) (12,43) |
| 159 | 3*53 | (5,23) (19,65) (10,37) (11,40) |
| 173 | 173 | (4,21) (23,78) |
| 179 | 179 | (1,15) (35,117) |
| 181 | 181 | (3,19) (27,91) |
| 183 | 3*61 | (2,17) (31,104) (7,29) (17,59) |
| 191 | 191 | (5,24) (22,75) |
| 199 | 199 | (9,35) (15,53) |
| 207 | 3*3*23 | (1,16) (38,127) (11,41) (13,47) |
| 211 | 211 | (3,20) (30,101) |
| 221 | 13*17 | (7,30) (20,69) |

## CHARACTERISTIC NUMBERS OF (3,1) SEQUENCES

| D | FACTORIZATION | SEQUENCES |
|---|---|---|
| 547 | 547 | (18,67) (21,76) |
| 549 | 3*3*61 | (1,25) (65,217) (19,70) (20,73) |
| 559 | 13*43 | (6,35) (45,152) |
| 563 | 563 | (2,27) (61,204) |
| 569 | 569 | (5,33) (49,165) |
| 571 | 571 | (3,29) (57,191) |
| 573 | 3*191 | (4,31) (53,178) (13,53) (29,101) |
| 597 | 3*199 | (1,26) (68,227) (7,38) (44,149) |
| 599 | 599 | (11,48) (34,117) |
| 601 | 601 | (15,59) (27,95) |
| 607 | 607 | (9,43) (39,133) |
| 621 | 3*3*3*23 | (5,34) (52,175) (17,65) (25,89) |
| 633 | 3*211 | (8,41) (43,146) (19,71) (23,83) |
| 641 | 641 | (13,54) (32,111) |
| 647 | 647 | (1,27) (71,237) |
| 653 | 653 | (7,39) (47,159) |
| 659 | 659 | (14,57) (31,108) |
| 663 | 3*13*17 | (2,29) (67,224) (11,49) (37,127) |
| 667 | 23*29 | (6,37) (51,172) (9,44) (42,143) |
| 673 | 673 | (3,31) (63,211) |
| 677 | 677 | (4,33) (59,198) |
| 689 | 13*53 | (29,102) (16,63) |
| 699 | 3*233 | (1,28) (74,247) (10,47) (41,140) |
| 701 | 701 | (17,66) (28,99) |
| 711 | 3*3*79 | (7,40) (50,169) (13,55) (35,121) |
| 719 | 719 | (19,72) (26,93) |
| 727 | 727 | (3,32) (66,221) |
| 729 | $3^6$ | (11,50) (40,137) |
| 731 | 17*43 | (5,36) (58,195) (22,81) (23,84) |
| 751 | 751 | (15,61) (33,115) |
| 753 | 3*251 | (1,29) (77,257) (8,43) (49,166) |
| 757 | 757 | (12,53) (39,134) |
| 771 | 3*257 | (2,31) (73,244) (7,41) (53,179) |
| 783 | 3*3*3*29 | (13,56) (38,131) (17,67) (31,109) |
| 789 | 3*263 | (4,35) (65,218) (5,37) (61,205) |
| 793 | 13*61 | (9,46) (48,163) |
| 797 | 797 | (11,51) (43,147) |
| 807 | 3*269 | (14,59) (37,128) (19,73) (29,103) |
| 809 | 809 | (1,30) (80,267) |
| 823 | 823 | (21,79) (27,97) |
| 829 | 829 | (15,62) (36,125) |
| 831 | 3*277 | (10,49) (47,160) (23,85) (25,91) |
| 841 | 29*29 | (3,34) (72,241) |
| 849 | 3*283 | (5,38) (35,122) (16,65) (64,215) |
| 857 | 857 | (13,57) (41,141) |
| 859 | 859 | (9,47) (51,173) |
| 867 | 3*17*17 | (1,31) (83,277) (11,52) (46,157) |
| 881 | 881 | (8,45) (55,186) |
| 883 | 883 | (18,71) (33,116) |

# TABLE OF COEFFICIENTS FOR THE EXPANSION OF

## $Y_{nk}$ IN TERMS OF $Y_k$ AND $Z_k$

Let there be two sequences whose terms are defined as follows

$$Y_n = \frac{r^n - s^n}{r - s} \quad \text{and} \quad Z_n = r^n + s^n$$

The expansion coefficients of $Y_{nk}$ in terms of $Y_k$ and $Z_k$ are as given in the following table. For example,

$$Y_{7k} = Y_k[Z_k^6 \;+5(-1)^{k+1}Z_k^4 \;+ 6Z_k^2 \;+(-1)^{k+1}]$$

| n | COEFFICIENTS | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | | | | | | |
| 2 | 1 | | | | | | |
| 3 | 1 | 1 | | | | | |
| 4 | 1 | 2 | | | | | |
| 5 | 1 | 3 | 1 | | | | |
| 6 | 1 | 4 | 3 | | | | |
| 7 | 1 | 5 | 6 | 1 | | | |
| 8 | 1 | 6 | 10 | 4 | | | |
| 9 | 1 | 7 | 15 | 10 | 1 | | |
| 10 | 1 | 8 | 21 | 20 | 5 | | |
| 11 | 1 | 9 | 28 | 35 | 15 | 1 | |
| 12 | 1 | 10 | 36 | 56 | 35 | 6 | |
| 13 | 1 | 11 | 45 | 84 | 70 | 21 | 1 |
| 14 | 1 | 12 | 55 | 120 | 126 | 56 | 7 |
| 15 | 1 | 13 | 66 | 165 | 210 | 126 | 28 |
| | 1 | | | | | | |
| 16 | 1 | 14 | 78 | 220 | 330 | 252 | 84 |
| | 8 | | | | | | |
| 17 | 1 | 15 | 91 | 286 | 495 | 462 | 210 |
| | 36 | 1 | | | | | |
| 18 | 1 | 16 | 105 | 364 | 715 | 792 | 462 |
| | 120 | 9 | | | | | |
| 19 | 1 | 17 | 120 | 455 | 1001 | 1287 | 924 |
| | 330 | 45 | 1 | | | | |
| 20 | 1 | 18 | 136 | 560 | 1365 | 2002 | 1716 |
| | 792 | 165 | 10 | | | | |
| 21 | 1 | 19 | 153 | 680 | 1820 | 3003 | 3003 |
| | 1716 | 495 | 55 | 1 | | | |

TABLE OF COEFFICIENTS FOR THE EXPANSION OF

$Y_{nk}$ IN TERMS OF $Y_k$ AND $Z_k$

| n | | | | | | | |
|---|---|---|---|---|---|---|---|
| 36 | 1 | 34 | 528 | 4960 | 31465 | 142506 | 475020 |
| | 1184040 | 2220075 | 3124550 | 3268760 | 2496144 | 1352078 | 497420 |
| | 116280 | 15504 | 969 | 18 | | | |
| 37 | 1 | 35 | 561 | 5456 | 35960 | 169911 | 593775 |
| | 1560780 | 3108105 | 4686825 | 5311735 | 4457400 | 2704156 | 1144066 |
| | 319770 | 54264 | 4845 | 171 | 1 | | |
| 38 | 1 | 36 | 595 | 5984 | 40920 | 201376 | 736281 |
| | 2035800 | 4292145 | 6906900 | 8436285 | 7726160 | 5200300 | 2496144 |
| | 817190 | 170544 | 20349 | 1140 | 19 | | |
| 39 | 1 | 37 | 630 | 6545 | 46376 | 237336 | 906192 |
| | 2629575 | 5852925 | 10015005 | 13123110 | 13037895 | 9657700 | 5200300 |
| | 1961256 | 490314 | 74613 | 5985 | 190 | 1 | |
| 40 | 1 | 38 | 666 | 7140 | 52360 | 278256 | 1107568 |
| | 3365856 | 7888725 | 14307150 | 20030010 | 21474180 | 17383860 | 10400600 |
| | 4457400 | 1307504 | 245157 | 26334 | 1330 | 20 | |

TABLE OF COEFFICIENTS FOR THE EXPANSION OF

$Z_{nk}$ IN TERMS OF $Z_k$

The use of these coefficients may be illustrated by the expansion of

$$Z_{7k} = Z_k^7 + 7(-1)^{k+1}Z_k^5 + 14Z_k^3 + 7(-1)^{k+1}Z_k$$

| n | COEFFICIENTS | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | | | | | | |
| 2 | 1 | 2 | | | | | |
| 3 | 1 | 3 | | | | | |
| 4 | 1 | 4 | 2 | | | | |
| 5 | 1 | 5 | 5 | | | | |
| 6 | 1 | 6 | 9 | 2 | | | |
| 7 | 1 | 7 | 14 | 7 | | | |
| 8 | 1 | 8 | 20 | 16 | 2 | | |
| 9 | 1 | 9 | 27 | 30 | 9 | | |
| 10 | 1 | 10 | 35 | 50 | 25 | 2 | |
| 11 | 1 | 11 | 44 | 77 | 55 | 11 | |
| 12 | 1 | 12 | 54 | 112 | 105 | 36 | 2 |
| 13 | 1 | 13 | 65 | 156 | 182 | 91 | 13 |
| 14 | 1 | 14 | 77 | 210 | 294 | 196 | 49 |
| | 2 | | | | | | |

TABLE OF COEFFICIENTS IN THE EXPANSION OF

$Z_{nk}$ IN TERMS OF $Z_k$

| n | COEFFICIENTS | | | | | | |
|---|---|---|---|---|---|---|---|
| 30 | 1 | 30 | 405 | 3250 | 17250 | 63756 | 168245 |
| | 319770 | 436050 | 419900 | 277134 | 119340 | 30940 | 4200 |
| | 225 | 2 | | | | | |
| 31 | 1 | 31 | 434 | 3627 | 20150 | 78430 | 219604 |
| | 447051 | 660858 | 700910 | 520676 | 260338 | 82212 | 14756 |
| | 1240 | 31 | | | | | |
| 32 | 1 | 32 | 464 | 4032 | 23400 | 95680 | 283360 |
| | 615296 | 980628 | 1136960 | 940576 | 537472 | 201552 | 45696 |
| | 5440 | 256 | 2 | | | | |
| 33 | 1 | 33 | 495 | 4466 | 27027 | 115830 | 361790 |
| | 834900 | 1427679 | 1797818 | 1641486 | 1058148 | 461890 | 127908 |
| | 20196 | 1496 | 33 | | | | |
| 34 | 1 | 34 | 527 | 4930 | 31059 | 139230 | 457470 |
| | 1118260 | 2042975 | 2778446 | 2778446 | 1998724 | 999362 | 329460 |
| | 65892 | 6936 | 289 | 2 | | | |
| 35 | 1 | 35 | 560 | 5425 | 35525 | 166257 | 573300 |
| | 1480050 | 2877875 | 4206125 | 4576264 | 3640210 | 2057510 | 791350 |
| | 193800 | 27132 | 1785 | 35 | | | |
| 36 | 1 | 36 | 594 | 5952 | 40455 | 197316 | 712530 |
| | 1937520 | 3996135 | 6249100 | 7354710 | 6418656 | 4056234 | 1790712 |
| | 523260 | 93024 | 8721 | 324 | 2 | | |
| 37 | 1 | 37 | 629 | 6512 | 45880 | 232841 | 878787 |
| | 2510820 | 5476185 | 9126975 | 11560835 | 10994920 | 7696444 | 3848222 |
| | 1314610 | 286824 | 35853 | 2109 | 37 | | |
| 38 | 1 | 38 | 665 | 7106 | 51832 | 273296 | 1076103 |
| | 3223350 | 7413705 | 13123110 | 17809935 | 18349630 | 14115100 | 7904456 |
| | 3105322 | 810084 | 128877 | 10830 | 361 | 2 | |
| 39 | 1 | 39 | 702 | 7735 | 58344 | 319176 | 1308944 |
| | 4102137 | 9924525 | 18599295 | 26936910 | 29910465 | 25110020 | 15600900 |
| | 6953544 | 2124694 | 415701 | 46683 | 2470 | 39 | |
| 40 | 1 | 40 | 740 | 8400 | 65450 | 371008 | 1582240 |
| | 5178240 | 13147875 | 26013000 | 40060020 | 47720400 | 43459650 | 29716000 |
| | 14858000 | 5230016 | 1225785 | 175560 | 13300 | 400 | 2 |