Combining (5.5) with Theorem (4.4) yields

(5.6) Theorem. Using concavity as a spy in a modified Fibonacci search is the optimal strategy for reducing the interval of uncertainty of concave functions.

6. FINAL REMARKS

From the proof of Theorem (5.6), it is apparent that the proposed search strategy for concave function is "min sup" rather than "min max." In other words, the problem is not well set. Indeed, it makes probably more sense for concave functions to decrease the uncertainty in the value of the minimum than in its location.

A similar argument as was used for proving (5.5) can be employed to show that for each $\epsilon > 0$ and each positive integer k there is a concave function for which the reduction of uncertainty by optimal search is improved by less than ϵ over unimodal search. In general, however, the improvement will be drastic, in particular if the function is well rounded, so to speak, and has a maximum in the interior.

REFERENCES

1. J. Boothroyd, "Fibonacci Search Algorithms in Theory and Practice," Unpublished paper, 1967.
2. S. M. Johnson, "Best Exploration for Maximum is Fibonaccian," the Rand Corporation, RM-1590, 1955.
3. J. Kiefer, "Sequential Minimax Search for a Minimum," Proc. Amer. Math. Soc., Vol. 4, pp. 502-506, 1963.
4. J. Kowalik and M. R. Osborne, Methods for Unconstrained Optimization Problems. Elsevier: New York - London - Amsterdam, 1968.
5. L. T. Oliver and D. J. Wilde, "Symmetrical Sequential Minimax Search for a Maximum," Fibonacci Quarterly, Vol. 2, 1964, pp. 169-175.
6. K. J. Overholt, An Instability in the Fibonacci and Golden Section Methods, Tidskrift for Informasjons Behandling, Vol. 5, 1965, p. 284.