POLYHEDRA, PENTAGRAMS, AND PLATO

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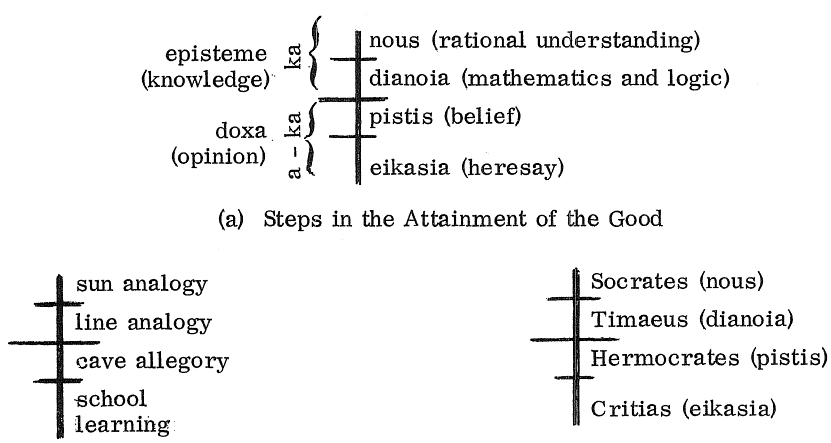
1. INTRODUCTION

The Divided Line

Plato believed that mathematics and logic were a necessary step in the pursuit of the Good, but that an extra step of enlightenment was necessary to achieve it. This is illustrated by the discussion of the divided line in the Republic. Plato divides a line into "unequal segments," [1] by first dividing the line (length a) into two unequal segments (ka and $a - ka$), the shorter one near the top. He then divides each segment by the same proportion k, and he labels the segments as shown in Fig. 1a. The divided line is one of four explanations of the Good offered in the Republic (Fig. 1b), where the line is itself an example of dianoia. Now, the problem with the line is that, following orders, one cannot construct the second and third segments unequal [2]. I leave the proof to the reader.

Platonists conclude that mathematics, symbolized by the line, while useful for describing the Good, contains inconsistencies reconciled only by a higher perception. In [2] is an excellent detailed explanation of the line.



(a) Steps in the Attainment of the Good

(b) Ways to Understand the Good

(c) Characters in the Timaeus

Fig. 1 The Divided Line

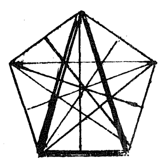
The irreconcilability of mathematics and the highest good was a particularly sore area for Plato. Irrational numbers were a case in point. Plato felt that, even though they were embodied in many beautiful objects (the Golden Ratio appeared in many buildings of his day), irrational numbers were without reason and impure.

2. THE DODECAHEDRON IN THE TIMAEUS

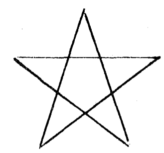
In this dialogue, Plato describes the celestial orbs as consisting of the five regular polyhedra, each of whose faces can be decomposed into the basic triangles which constitute matter [3]. He divides them up as shown in Fig. 2. The Pythagoreans divided the pentagonal faces of the dodecahedron into 30 elementary scalene triangles [4], as shown in Fig. 3a.

POLYHEDRON	FACE	ELEMENTARY TRIANGLE
pyramid	(4) triangles	
octahedron	(8) triangles	(6) 30°-60°-90°
icosahedron	(20) triangles	
cube	(6) squares	(4) 45°-45°-90°
dodecahedron	(12) pentagons	Plato does not divide these.

Fig. 2 The Celestial Orbs and their Constituent Triangles, Squares, and Pentagons



(a) Decomposed into elementary triangles



(b) Represented as a pentagram

Fig. 3 The Pentagonal Face of a Dodecahedron [4]

This decomposition provides the outline of the famous pentagram (Fig. 3b), the Pythagorean symbol of recognition, meaning "Health" [5]. The heavy outline in Fig. 3a marks a 72°-72°-36° isosceles triangle, the ratio of whose sides is the Golden Ratio, which is irrational [6].

The first four polyhedra describe the Sun, the Moon, and planets [7], and comprise collectively the Circle of the Different; but the dodecahedron, the Circle of the Same, is the celestial sphere itself. The twelve faces of the dodecahedron are the twelve signs of the Zodiac [8]. Where the other orbs rotate at various intervals, the dodecahedron rotates exactly once each day (actually the rotation of the earth). Plato gives the dodecahedron special compliments. Because of its diurnal regularity, it has Sameness and Supremacy and is Self-Moving, quite a nice Platonic praise. Most importantly, the dodecahedron is rational. He says:

> Now whenever discourse that is alike true... is about that which is sensible, and the circle of the Different, moving aright, carries its message throughout all its soul--then there arise judgments and beliefs that are sure and true. But whenever discourse is concerned with the rational, and the circle of the Same, running smoothly, declares it, the result must be rational understanding and knowledge [9].

Plato contradicts himself. At the root of the dodecahedron is the Golden Ratio, which is irrational and Platonically imperfect; yet Plato describes the dodecahedron as rational and perfect.

The easiest explanation of this contradiction is that it is a Platonic aberration. But I think that Plato knew it all along, and that it is an attempt to show a flaw in Timaeus' argument.

3. INTERPRETATION

Plato is giving mathematicians a subtle lesson on the limits of their perceptions. It is not Plato speaking in this dialogue, it is Timaeus--in fact, there are four speakers: Critias, Hermocrates, Timaeus, and Socrates. This is an important Platonic clue. The characters might be ranked as shown in Fig. 1c. Critias begins with a story about Atlantis which he heard from the old Critias, who heard it from Solon, who heard it from the priest of an ancient Egyptian province (an example of heresay). Hermocrates is the one who introduces the story to Plato (belief). Timaeus is the scientist, describing the universe with natural laws and mathematics as he sees them (mathematics). Plato (rational understanding) never gets the last word.

Timaeus' discussion is a model of dianoia. We Timaeuses might describe the world in our mathematical terms and point to the beauty of our models, but according to Plato, our models have built-in contradictions. Like the Line, the Timaeus is a mathematical description of nature; and like the Line, it must contain hidden contradictions and imperfections. Timaeus' mathematical construct, the dodecahedron, is superficially beautiful and rational, but it contains hidden the Golden Ratio and its imperfect, irrational $\sqrt{5}$.

Now most of us probably do not see anything wrong with $\sqrt{5}$; after all, it's much neater than e or i. And I personally think mathematics is quite beautiful. But Plato believed that mathematics cannot simultaneously retain its simplicity and achieve beauty, that mathematics alone is insufficient to achieve the Good, and that the Golden Ratio is the paradigm of mathematics' aesthetic inadequacy, as shown by the dodecahedron.

We Fibonacci lovers can at least savor the knowledge that the great Greek genius spent so much time thinking about one of our favorite numbers.

REFERENCES

1. Plato, Republic: VI, Paul Shorey (trans.), The Collective Dialogues of Plato, E. Hamilton and E. Cairns (eds.), Princeton: Princeton University Press, 1961, 509d.
2. Robert S. Brumbaugh, Plato's Mathematical Imagination, Bloomington: Indiana University Press, 1954, p. 98.
3. Plato, Timaeus, Francis M. Cornford (trans.), Indianapolis: Bobbs-Merrill Co., 1959, 54d-55c.
4. Thomas L. Heath, The Thirteen Books of Euclid's Elements, New York: Dover Publications, Inc., 1956, Vol. 2, p. 98.
5. Ibid., pp 99. This is the answer to word "K" in Marjorie Bicknell, "A Fibonacci Crostic," Fibonacci Quarterly, Vol. 9, No. 5, Dec. 1971, pp. 538-540. The pentagram is the emblem of the Fibonacci Association (see cover).
6. According to legend, Hippasus was struck down at sea for discovering the dodecahedron and thus introducing irrational numbers to the world.
7. Kepler published a model of these in Mysterium cosmographicum in 1595.
8. Plato, Timaeus, op. cit., 55c.
9. Ibid, 37b-c. The underlines are mine.