We have therefore

$$F_{a^k}(x) = F_{a^{k+1}}(x) + (x^{2^{k+1}} + x^{u_{k+2}})F_{a^{k+2}}(x) . \tag{6.17}$$

Now

$$B(x) = x^2 F_{a^2}(x) ,$$

so that $F_{a^2}(x)$ is rationally related to $A(x) = F_a(x)$. Then by (6.17) the same is true of $F_{a^2}(x)$ and so on.

We may state

Theorem 6.3. For arbitrary w, the function $F_w(x)$ is rationally related to $A(x)$, that is, there exist polynomials $P_w(x)$, $Q_w(x)$, $R_w(x)$ such that

$$P_w(x)F_w(x) = Q_w(x)A(x) + R_w(x) .$$

It seems plausible that $A(x)$ and $D_1(x)$ are not rationally related but we have been unable to prove this.

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it is clear that we have proved (5).

As for (2), we have

$$aL_n - L_{n+1} = b^n(a - b) = b^n\sqrt{5} .$$

For $n \geq 4$

$$\left| b^n\sqrt{5} \right| \leq b^4\sqrt{5} = \tfrac{1}{2}(7 - 3\sqrt{5})\sqrt{5} < \tfrac{1}{2} .$$

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1. R. Anaya and J. Crump, "A Generalized Greatest Integer Function Theorem," Fibonacci Quarterly, Vol. 10 (1972), pp. 207-211.