CORRIGENDUM TO: ENUMERATION OF TWO-LINE ARRAYS

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The proof of (2.5) and (2.7) in the paper: "Enumeration of Two-Line Arrays" [1] is incorrect as it stands. A corrected proof follows.

Let $g(n,k)$ denote the number of two-line arrays of positive integers

$$\boxed{\begin{matrix} a_1 & a_2 & \cdots & a_n \\ b_1 & b_2 & \cdots & b_n \end{matrix}}$$

satisfying the inequalities

$$\begin{array}{ll} \max(a_i,b_i) \leqslant \min(a_{i+1}, b_{i+1}) & (1 \leqslant i < n), \\ \max(a_i,b_i) \leqslant i & (1 \leqslant i < n) \end{array}$$

and

$$\max(a_n, b_n) = k.$$

We wish to show that

$$g(n+k,k) = \sum_{j=1}^{k} g(j,j)g(n+k-j,\, k-j+1) \qquad (n \geqslant 1). \tag{2.7}$$

Let j be the greatest integer $\leqslant k$ such that

$$\max(a_j,b_j) = j.$$

It follows that $a_{j+1} = b_{j+1} = j$.

Consider the array

$$\begin{array}{|ccc|ccc|} a_1 & \cdots & j & j & \cdots & k \\ 1 & \cdots & \bullet & j & \cdots & \bullet \end{array}$$

Put

$$\left.\begin{array}{l} a'_i = a_{j+i} - (j-1) \\ b'_i = b_{j+i} - (j-1) \end{array}\right. \qquad (1 \leqslant i \leqslant n+k-j).$$

It follows from the conditions satisfied by a_i, b_i that

$$\begin{array}{ll} \max(a'_i, b'_i) \leqslant \min(a'_{i+1}, b'_{i+1}) & (1 \leqslant i < n+k-j), \\ \max(a'_i, b'_i) \leqslant i & (1 \leqslant i \leqslant n+k-j), \\ \max(a'_{n+k-j}, b'_{n+k-j}) = k-j+1. & \end{array}$$

This evidently yields (2.7).

REFERENCE

1. L. Carlitz, "Enumeration of Two-Line Arrays," *The Fibonacci Quarterly,* Vol. 11, No. 2 (April 1973), pp. 113–130.

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