$$L(3^n) = \prod_{k=0}^{n-1} \frac{L(3^{k+1})}{L(3^k)} = \prod_{k=0}^{n-1} [L(2\cdot 3^k) + 1].$$

Also solved by Paul S. Bruckman, Herta T. Freitag, Ralph Garfield, Graham Lord, C.B.A. Peck, M.N.S. Swamy, Gregory Wulczyn, and the Proposer.

REGULAR POLYGON RELATION

B-267 Proposed by Marjorie Bicknell, Wilcox High School, Santa Clara, California.

Let a regular pentagon of side p, a regular decagon of side d, and a regular hexagon of side h be inscribed in the same circle. Prove that these lengths could be used to form a right triangle; i.e., that $p^2 = d^2 + h^2$.

Solution by Gregory Wulczyn, Bucknell University, Lewisburg, Pennsylvania.

Hobson, in *Plane and Advanced Trigonometry,* on page 31 states:

$$\sin 18^\circ = \frac{\sqrt{5}-1}{4}, \qquad \sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}.$$

$$p = 2r \sin 36^\circ, \qquad h = r, \qquad d = 2r \sin 18^\circ$$

$$h^2 + d^2 = r^2 + \frac{4r^2}{16}(6 - 2\sqrt{5}) = \frac{(5-\sqrt{5})}{2} r^2$$

$$p^2 = \frac{4r^2}{16}(10 - 2\sqrt{5}) = \frac{5-\sqrt{5}}{2} r^2$$

$$\therefore\ p^2 = h^2 + d^2 .$$

Also solved by Paul S. Bruckman, Warren Cheves, Herta T. Freitag, Graham Lord, C.B.A. Peck, Paul Smith, M.N.S. Swamy, David Zeitlin, and the Proposer.

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and in order for (ii) to be satisfied a must equal 1. Therefore, the given sequence must be the Fibonacci sequence.

NOTE: The most general sequence satisfying (i) has the form

$$\cdots, ax_1, x_1, x_0 = 0, x_1, ax_1, (a^2+1)x_1, \cdots.$$

Also, if condition (ii) is weakened to the restriction that two consecutive terms be relatively prime, then the most general sequence would have the form

$$\cdots, -a, 1, x_0 = 0, 1, a, a^2+1, \cdots.$$

REFERENCE

1. V.E. Hoggatt, Jr., *Fibonacci and Lucas Numbers,* Houghton-Mifflin Co., New York, 1969.

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