Thus the generating function for $u_r(k,m,n)$ is

$$\prod_{j=1}^{n} \left(\sum_{h=0}^{r} q^{hj} x^h \right) . \tag{3.3}$$

But it is well known (see for example [3, p. 10] for $r = 1$) that the generating function for $v_r(k,m,n)$ is also (3.3). Hence we have (3.1). This identity is also evident from the Ferrers graph.

To illustrate (3.1) and (3.2) let $m = 7$, $n = 4$, $k = 3$ and $r = 2$. The sequences enumerated by $f_3(7,5;\,2,0)$ are $0,0,1,3,3$, $0,0,2,2,3$ and $0,1,1,2,3$. The function $u_2(3,7,4)$ counts the corresponding partitions, namely 13^2, 2^23 and 1^223. The partitions which $v_2(3,7,4)$ enumerates are 2^23, 13^2 and 124. From the graphs

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. . .      . . .      . . . .
. .        . . .      . .
. .        .          .
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we observe that 13^2 is the conjugate of 2^23, 2^23 is the conjugate of 13^2 and 1^223 is the conjugate of 124.

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SPECIAL CASES

Putting $r = 1$, $s = 0$, we obtain the generating function for the Fibonacci sequence (see [3] and Riordan [6]). Putting $r = 2$, $s = -1$, we obtain the generating function for the Lucas sequence (see [3] and Carlitz [1]).

Other results in Riordan [6] carry over to the H-sequence. The H-sequence (and the Fibonacci and Lucas sequences), and the generalized Fibonacci and Lucas sequences are all special cases of the W-sequence studied by the author in [4]. More particularly,

$$\{H_n\} = \{w_n(r, r+s;\, 1, -1)\}$$

and so

$$\{f_n\} = \{w_n(1, 1;\, 1, -1)\}, \qquad \{a_n\} = \{w_n(2, 1;\, 1, -1)\} .$$

Interested readers might consult the article by Kolodner [5] which contains material somewhat similar to that in [3], though the methods of treatment are very different.

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