of the first k odd primes, we see that $k = 1$ is the lowest k for which

$$2^k k! < \prod_{i=1}^{k} p_i .$$

But once this inequality holds for one k, it holds for all larger k. For by multiplying each side by $2(k + 1)$, we get

$$2^{k+1}(k+1)! < \prod_{i=1}^{k} p_i \cdot 2(k+1) < \prod_{i=1}^{k+1} p_i ,$$

since $p_{k+1} > 2(k + 1)$.

Therefore, for all k,

$$a_k < \prod_{i=1}^{k} p_i ,$$

and in particular, a_k is less than any product of k distinct odd primes. We conclude that no product of distinct odd primes can be super-perfect, and the theorem follows.

****

SIGNIFICANCE OF EVEN-ODDNESS OF A PRIME'S PENULTIMATE DIGIT

WILLIAM RAYMOND GRIFFIN
Dallas, Texas

By elementary algebra one may prove a remarkable relationship between a prime number's penultimate (next-to-last) digit's even-oddness property and whether or not the prime, p, is of the form $4n + 1$, or $p \equiv 1 \pmod 4$, or of the form $4n + 3$, or $p \equiv 3 \pmod 4$, where n is some positive integer.

The relationships are as follows:

A. Primes $\equiv 1 \pmod 4$

(1) If the prime, p, is of the form $10k \pm 1$, k being some positive integer, then the penultimate digit is *even*.

(2) If p is of the form $10k \pm 3$, then the penultimate digit is *odd*.

B. Primes $\equiv 3 \pmod 4$

(1) If p is of the form $10k \pm 1$, then the penultimate digit is *odd*.

(2) If p is of the form $10k \pm 3$, then the penultimate digit is *even*.

The beauty of these relationships is that, by inspection *alone*, one may instantly observe whether or not a prime number is $\equiv 1$, or $\equiv 3 \pmod 4$. These relationships are especially valuable for very large prime numbers—such as the larger Mersenne primes.

Thus, it is seen from inspection of the penultimate digits of the Mersenne primes, as given in [1], that all of the given primes are $\equiv 3 \pmod 4$. This holds true for *all* Mersenne primes, however large they may be, for, by adding and subtracting 4 from $M_p = 2^p - 1$ and re-arranging, we have

$$M_p = 2^p - 1 + 4 - 4 = 2^p - 4 + 3 = 4(2^{p-2} - 1) + 3 \equiv 3 \pmod 4.$$

[Continued on Page 208.]