$p \equiv 13$ or $17 \pmod{20}$: The residue ½ does not appear. Exactly one square root of -1 appears.

$p \equiv 1$ or $9 \pmod{20}$ and $\beta(p) = 1$ or 2: The residue ½ appears. Both square roots of -1 and the residues

$$\frac{1 \pm \sqrt{5}}{2} \pmod{p}$$

do not appear.

$p \equiv 1$ or $9 \pmod{20}$ and $\beta(p) = 4$: The residues ½ and

$$\frac{1 \pm \sqrt{5}}{2} \pmod{p}$$

do not appear. Exactly one square root of $-1 \pmod{p}$ appears.

REFERENCE

1. John H. Halton, "On the Divisibility Properties of Fibonacci Numbers," *The Fibonacci Quarterly,* Vol. 4, No. 3 (Oct. 1966), pp. 217–241.

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[Continued from P. 321.]

If $(a,b) = 1$, then

$$(a^2/b^2) = 1,$$
$$(-a^2/b^2) = 1,$$
$$(a^2/-b^2) = 1,$$
$$(-a^2/-b^2) = -1;$$

$$(a/b^2) = 1,$$
$$(-a/b^2) = 1,$$
$$(a/-b^2) = (a/-1),$$
$$(-a/-b^2) = -(a/-1);$$

$$(a^2/b) = 1,$$
$$(-a^2/b) = (-1/b),$$
$$(a^2/-b) = 1,$$
$$(-a^2/-b) = -(-1/b);$$

$$(a/b) = (a/b),$$
$$(-a/b) = (a/b)(-1/b),$$
$$(a/-b) = (a/b)(a/-1),$$
$$(-a/-b) = -(a/b)(a/-1)(-1/b).$$

It remains to evaluate $(a/-1)$. Since $(-a^2/-1) = -1$, therefore $(a/-1) = -(-a/-1)$. This means that $(a/-1)$ cannot be defined in terms of an integer. Either $(a/-1) = 1$ if and only if a is positive or $(a/-1) = 1$ if and only if a is negative. The choice of alternative is dictated by the fact that $(1/-1) = 1$ and $(-1/-1) = -1$. Therefore, $(a/-1) = 1$ if and only if a is positive.

(See Tables 1 through 4.)

[Continued on P. 328.]