VARIATION IN THE NUMBER OF RAY- AND DISC-FLORETS IN FOUR SPECIES OF COMPOSITAE

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INTRODUCTION

The compositae is one of the largest families of the vascular plants, comprising about 1000 genera and about 30,000 species. The members of this family are distributed in almost all parts of the world and are readily recognized by their unique disc-shaped inflorescence, composed of numerous pentamerous florets packed on an involucrate head.

There is a good deal of variation in the numbers of ray-florets and disc-florets in many compositae. Moreover, beautiful phyllotactic configurations become visible due to the unique arrangement of florets/fruits in the head. Definite equiangular spirals appear on the head of a composite, which run either right-handed (counter-clockwise) or left-handed (clockwise). Another different set of spirals run opposite to the former spirals, and these intersect each other, such as: 2/3, 3/5, 5/8, 8/13, 13/21, 21/34, ···. The numerators or the denominators of this series, when considered alone, form the successive stages of the famous Fibonacci Sequence.

In this note are presented the results of a study on the variation in the number of ray-florets in four species of compsoitae and also the variation in the number of disc-florets in one species.

PRESENTATION OF DATA

Variation in the number of ray-florets: Data on the variation in the number of ray-florets were obtained on:

1. *Tridax procumbens*
2. *Cosmos bipinnatus* (two varieties)
3. *Coreopsis tinctoria*
4. *Helianthus annuus.*

Tridax procumbens is an annual weed which usually grows on roadsides and wastelands. Its capitulum is small, the diameter of a head usually measuring 4.5 mm to 5.5 mm and raised on a long peduncle. *Cosmos bipinnatus* is a common flower which grows during winter and is available in white, pink and saffron colours. *Coreopsis tinctoria* and *Helianthus annuus* are also common in India. The head of *Coreopsis* is small, but that of *Helianthus* is quite large, with bright yellow ray-florets.

Data on *Tridax procumbens* were obtained from a locality near Andhra University, Waltair, Andhra Pradesh, India, during July 1972. In all, 4000 heads were observed.

Data on 300 heads of each of the two varieties of *Cosmos bipinnatus,* the saffron coloured variety and the white and pink coloured variety, were gathered from two localities at Calcutta. In both the localities, nearly 100 p.c. of the heads possessed eight rays each.

Data on 500 heads of *Coreopsis tinctoria* were collected from the gardens of Royal Agri-Horticultural Society, Calcutta.

One thousand and two heads of *Helianthus annuus* were observed in three different localities in Calcutta.

Variation in the number of disc-florets: Data on the variation in the number of disc-florets were collected on two varieties of *Cosmos bipinnatus,* the saffron coloured variety (Variety 1) and the white and pink coloured variety (Variety 2) from two localities in Calcutta.

Thirty heads of Variety 1 and 61 heads of Variety 2 were gathered from the gardens of the Indian Statistical Institute, Calcutta, and the number of disc-florets on each of the heads were counted.

DISCUSSION

From the data represented in Fig. 1, it is seen that though there is a great deal of variation in the numbers of ray-florets per head within the same species, the mode of each species invariably turns out to be a Fibonacci number. For *Tridax* the mode is at the fifth Fibonacci number; that is, at 5; for both varieties of *Cosmos* as well as for *Coreopsis* the mode is at the sixth Fibonacci number; that is, at 8; and for *Helianthus* the mode is at the eighth Fibonacci number; that is, at 21. Among the four species of compositae observed, the variation in the number of ray-florets is greatest for *Helianthus* and least for *Cosmos.*

Such variation surely has a genetic component and some (Ludwig, 1897) believed that (both within and between plants) it is largely a result of climatic factors and nutrition. These multimodal distributions are not totally new, and were demonstrated by Ludwig in the ray-florets of *Bellis perennis,* disc-florets in *Achilles millefolium* and flowers in the umbels of *Primula veris* as early as in 1890 (Briggs and Walters, 1969).

Such modal variation can be explained by the model suggested by Turing (1952). Turing considered a system of chemical substances, or "morphogens," reacting together and diffusing through a tissue. He showed that such a system, though originally homogeneous, may later develop a pattern or a structure due to instability of the homogeneous equilibrium. In the simple case of an isolated ring of cells, one form of instability gives rise to a standing wave of concentration of the morphogens. For any given set of values of the constants for the rates of reaction and diffusion there will be a "chemical wave-length" of β. If the circumference of the ring, s, is divided by β, the result will not usually be an integer. Yet the system necessarily forms an integral number of waves, typically the integer nearest to s/β.

This provides a simple model of the process whereby an integral number of discrete structures can arise from a homogeneous tissue. All individuals of a population will have n structures if:

$$n - (1/2) < s/\beta < n + (1/2).$$

Writing Var. (s), Var. (β), and Var. (s/β) for the variances of s, β, and s/β respectively, and Cov. (s,β) for the covariance between s and β, one can easily see that:

$$\text{Var.}\,(s/\beta) = \beta^{-4}[\beta^2\cdot\text{Var.}\,(s) + s^2\cdot\text{Var.}\,(\beta) - 2s\beta\cdot\text{Cov.}\,(s,\beta)]\,.$$

It is also interesting to know why the modes in the distribution of heads of ray-florets turn out to be Fibonacci numbers. An explanation which seems logical to us is the following:

The general formula for obtaining the Fibonacci numbers is:

$$F_n = F_{n-1} + F_{n-2},\; n = 3, 4, 5, \cdots; \qquad F_1 = F_2 = 1,$$

where F_n denotes the n^{th} Fibonacci number. When n is large, we can write:

$$F_n \doteq (1/\sqrt{5})\cdot[(1+\sqrt{5})/2]^n\,,$$

(where $\doteq$ means "approximately equal to"), and one can approximate it by the continuous curve:

$$y = (1/\sqrt{5})\cdot[(1+\sqrt{5})/2]^x\,,$$
$$= 0.4472 \times (1.6180)^x\,.$$

For all practical purposes, the Fibonacci numbers lie on this curve in its higher stages, and moreover it represents perfect exponential growth; presumably tending to reduce the size of the florets to the optimum necessary for quick production of an adequate number of single seeded fruits. So the appearance of the Fibonacci numbers as the modes for the distribution of the ray-florets on the heads can be taken as an indication for perfect growth, as is usually the case. Also it is well known that Fibonacci phyllotaxis give optimum illumination to the photo-synthetic surfaces of plants (Davis, 1971).

The appearance of the Fibonacci numbers can also be explained on the consideration that the individual flowers emerge at a uniform speed at fixed intervals of time along a logarithmic spiral, $r = e^{a\alpha}$ with small a and with an initial angle α_1 = 137.5° (Mathai and Davis, 1974). They also show that the above logarithmic spiral may be a natural outcome of the supply of genetic material in the form of pulses at constant intervals of time and obeying the law of fluid flow.

A new theory which has been proposed by Leppik (1960) has emerged from studies of the behaviour of pollinating insects and their relationships with flowers. On the basis of numerous observations and behaviour tests, Leppik

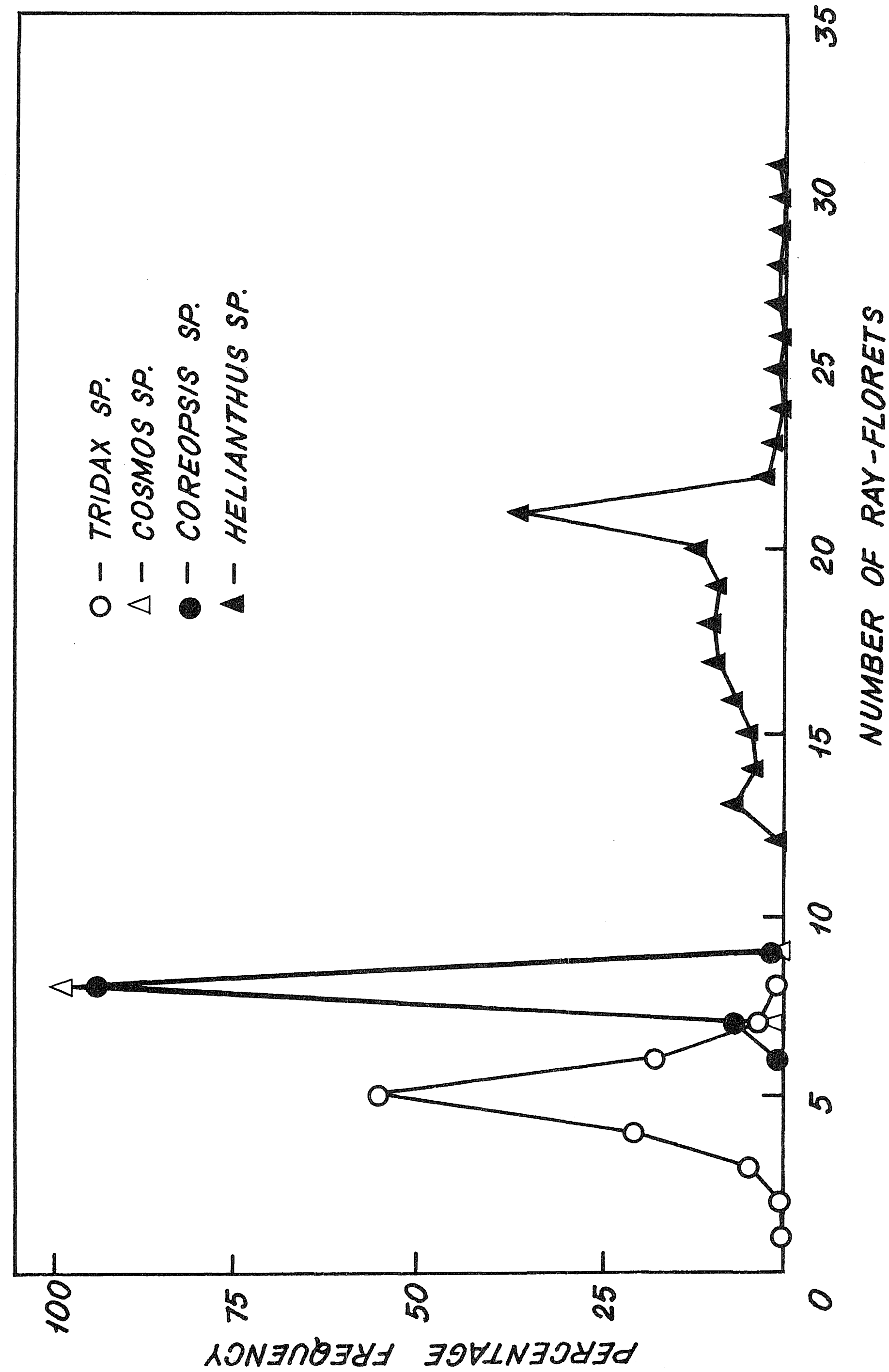
○ – TRIDAX SP.
△ – COSMOS SP.
● – COREOPSIS SP.
▲ – HELIANTHUS SP.
0
5
10
15
20
25
30
35
NUMBER OF RAY-FLORETS
0
25
50
75
100
PERCENTAGE FREQUENCY

Figure 1

has ascertained that most pollinating insects have the ability to distinguish floral characteristics—angular-form and radial-symmetry in particular. He has hypothesized that some numeral patterns, which include the Fibonacci numbers, are more symmetrically arranged than others. And hence, floral differentiation occurred and this has gradually led to the evolution of ecotypes with specific numeral patterns.

As we have already seen, in *Cosmos,* there is no (negligible) variation in the number of ray-florets and this turns out to be 8. But there is a great variation in the number of disc-florets. This shows that the correlation between the number of ray-florets and the number of disc-florets is almost zero. Also another interesting fact noticed is that there is no correlation between the size of a flower (when the head is looked at as a single flower) and the number of disc-florets present on the flower-head.

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