

Method XI. For yet another method see A. G. Shannon's solution in the April 1976 *Advanced Problem Section* solution to H-237.

REFERENCES

1. I. J. Good, "A Reciprocal Series of Fibonacci Numbers," *The Fibonacci Quarterly*, Vol. 12, No. 4 (Dec. 1974), p. 346.
2. D. A. Millin, Problem H-237, *The Fibonacci Quarterly*, Vol. 12, No. 3 (Oct. 1974), p. 309.
3. V. E. Hoggatt, Jr., *Fibonacci and Lucas Numbers*, Houghton-Mifflin, Boston, 1969.
4. I. D. Ruggles, "Some Fibonacci Results Using Fibonacci-Type Sequences," *The Fibonacci Quarterly*, Vol. 1, No. 2 (April 1963), pp. 75-80.
5. Ken Siler, "Fibonacci Summations," *The Fibonacci Quarterly*, Vol. 1, No. 3 (Oct. 1963), pp. 67-69.
6. V. E. Hoggatt, Jr., and Marjorie Bicknell, "A Reciprocal Series of Fibonacci Numbers with subscripts $k2^n$," *The Fibonacci Quarterly*, to appear.
7. I. J. Good and P. S. Bruckman, "A Generalization of a Series of De Morgan with Applications of Fibonacci Type," *The Fibonacci Quarterly*, Vol. 14, No. 3 (Oct. 1976), pp. 193-196.
8. L. Carlitz, private communication.

[Continued from Page 253.]

Then the sequence

$$(w_n) = (\log H_n H_n^*)$$

is u.d. mod 1.

Proof. We have

$$w_{n+1} - w_n = \log \frac{H_{n+1}}{H_n} + \log \frac{H_{n+1}^*}{H_n^*},$$

which tends to

$$2 \log \frac{1 + \sqrt{5}}{2}$$

as $n \rightarrow \infty$ for

$$\frac{H_{n+1}}{H_n} = \frac{qF_n + pF_{n-1}}{qF_{n-1} + pF_{n-2}} = \frac{q(F_n/F_{n-1}) + p}{q(F_{n-1}/F_{n-2}) + p} \cdot \frac{F_{n-1}}{F_{n-2}}$$

goes to

$$\frac{1 + \sqrt{5}}{2}$$

as $n \rightarrow \infty$

Theorem 3. Let p, q, p^*, q^*, H_n and H_n^* have the same meaning as in Theorem 2. Then the sequence

$$(x_n) = (\log (H_n + H_n^*))$$

is u.d. mod 1.

Proof. By the definitions of H_n and H_n^* we have

$$H_n + H_n^* = (q + q^*)F_{n-1} + (p + p^*)F_{n-2} \quad (n \geq 3)$$

and so we see that

$$x_{n+1} - x_n = \log \left(\frac{(H_{n+1} + H_{n+1}^*)}{(H_n + H_n^*)} \right) = \log \frac{(q + q^*)F_n + (p + p^*)F_{n-1}}{(q + q^*)F_{n-1} + (p + p^*)F_{n-2}}$$

[Continued on Page 281.]