$$\sum_{k=0}^{n} F_k F_{k+2m+1} = F_n F_{n+2m+2}.$$

If n is odd, we have

$$\sum_{k=0}^{n} F_k F_{k+2m+1} = F_{n-1}F_{n+2m+1} + F_n F_{n+2m+1} = F_{n+1}F_{n+2m+1}.$$

Also solved by the sovlers of B-320.

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[Continued from page 469.]

ADVANCED PROBLEMS AND SOLUTIONS

Show that

$$L(m,n) - 25F(m,n) = 8L_{m+n}F_{m+1}F_{n+1}.$$

Solution by the Proposer.

It follows from the Binet formulas

$$F_m = \frac{\alpha^m - \beta^m}{\alpha - \beta}, \qquad L_m = \alpha^m + \beta^m$$

that

$$5F_m F_n = L_{m+n} - (\alpha^m\beta^n + \alpha^n\beta^m),$$

so that

$$5F_{i+j}F_{m-i+n-j} = L_{m+n} - (\alpha^{i+j}\beta^{m-i+n-j} + \alpha^{m-i+n-j})$$
$$5F_{i+n-j}F_{m-i+j} = L_{m+n} - (\alpha^{i+n-j}\beta^{m-i+j} + \alpha^{m-i+j}\beta^{i+n-j}).$$

Hence

$$25F_{i+j}F_{m-i+j}F_{i+n-j}F_{m-i+n-j} = L_{m+n}^2 - L_{m+n}(\alpha^{i+j}\beta^{m-i+n-j} + \alpha^{m-i+n-j}\beta^{i+j} + \alpha^{i+n-j}\beta^{m-i+j} + \alpha^{m-i+j}\beta^{i+n-j})$$
$$+ (\alpha^{2i+n}\beta^{2m-2i+n} + \alpha^{2m-2i+n}\beta^{2i+n} + \alpha^{m+2j}\beta^{m+2n-2j} + \alpha^{m+2n-2j}\beta^{m+2j}).$$

It follows that

$$25F(m,n) = (m+1)(n+1)L_{m+n}^2 - 4L_{m+n}F_{m+1}F_{n+1} + 2(-1)^n(n+1)F_{2m+2} + 2(-1)^m(m+1)F_{2n+2}.$$

Similarly,

$$L(m,n) = (m+1)(n+1)L_{m+n}^2 + 4L_{m+n}F_{m+1}F_{n+1} + 2(-1)^n(n+1)F_{2m+2} + 2(-1)^m(m+1)F_{2n+2}.$$

Therefore,

$$L(m,n) - 25F(m,n) = 8L_{m+n}F_{m+1}F_{n+1}.$$

Also solved by P. Bruckman.

EDITORIAL REQUEST! Send in your problem proposals!

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