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LETTER TO THE EDITOR

March 20, 1974

Dear Sir:

I would like to contribute a note, letter, or paper to your publication expanding the topic presented below.

Following is a sequence of right triangles with integer sides, the smaller angles approximating 45 degrees as the sides increase:

(1) 3, 4, 5, – 21, 20, 29 – 119, 120, 169 – ⋯.

Following is another sequence of such "Pythagorean" triangles, the smallest angle approximating 30 degrees as the sides increase:

(2) 15, 8, 17 – 209, 120, 241 – 2911, 1680, 3361 – 23408, 40545, 46817 – 564719, 326040, 652081 ⋯

The scheme for generating these sequences resembles that for generating the Fibonacci sequence 1, 2, 3, 5, and so on.

Let g_k and g_{k-1} be any two positive integers, $g_k > g_{k-1}$. Then, as is well known,

(3) $$g_k^2 - g_{k-1}^2, \qquad 2g_kg_{k-1}, \qquad \text{and} \qquad g_k^2 + g_{k-1}^2$$

are the sides of a Pythagorean triangle.

Now let m and n be two integers, non-zero, and let

(4) $$g_{k+1} = ng_k + mg_{k-1}$$

to create a sequence of g's.

If $g_1 = 1,\ g_2 = 2,\ m = 1,\ n = 2,$ substitution in (4) and (3) gives the triangle sequence in (1) above.

If $g_1 = 1,\ g_2 = 4,\ m = -1,\ n = 4,$ the resulting triangle sequence is (2) above.

If the Fibonacci sequence itself is used $(m = n = 1)$, a triangle sequence results in which the ratio between the short sides approximates 2:1.

In general, it is possible by this means to obtain a sequence of Pythagorean triangles in which the ratio of the legs, or of the hypotenuse to one leg, approximates any given positive rational number p/q (p and q positive non-zero integers, $p \geqslant q$). It is easy to obtain m and n and good starting values g_1 and g_2 given p/q, and there is more to the topic besides, but I shall leave all that for another communication.

For all I know, this may be an old story, known for centuries.

However, Waclaw Sierpinski, in his monograph *Pythagorean Triangles* (Scripta Mathematica Studies No. 9, Graduate School of Science, Yeshiva University, New York, 1962), does not give this method of obtaining such triangle sequences, unless I missed it in a hasty reading. He obtains sequence (1) above by a different method (Chap. 4). He shows also how to obtain Pythagorean triangles having one angle arbitrarily close to any given angle in the first quadrant (Chap. 13); but again, the method differs from the one I have outlined.

[Continued on page 10.]