sequence S_1, the first divisors of successive columns were 1, 2, 6, 20, 70, ⋯, the central column of Pascal's triangle which gave rise to the Catalan numbers originally. For S_2, they are 1, 4, 21, 120, ⋯, which diagonal of Pascal's triangle yields S_2 upon successive division by $(3j+1)$, $j = 0, 1, 2, \cdots$, and $S_2^2 = \{1, 2, 7, 60, \cdots\}$ upon successive division by 1, 2, 3, 4, ⋯. For S_3, the first divisors are 1, 6, 45, ⋯, which produce $S_3^3 = \{1, 3, 15, 91, \cdots\}$, upon successive division by 1, 2, 3, 4, ⋯.

REFERENCES

1. V. E. Hoggatt, Jr., and Marjorie Bicknell, "Catalan and Related Sequences Arising from Inverses of Pascal's Triangle Matrices," *The Fibonacci Quarterly*, Vol. 14, No. 5, pp. 395–404.
2. V. E. Hoggatt, Jr., and Marjorie Bicknell, "Pascal, Catalan, and General Sequence Convolution Arrays in a Matrix," *The Fibonacci Quarterly*, Vol. 14, No. 2, pp. 136–142.

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ON THE N CANONICAL FIBONACCI REPRESENTATIONS OF ORDER N

$$x^N - \sum_{i=0}^{N-1} x^i$$

for some $N \geq 2$. Then

$$a^{N+i} = \sum_{k=0}^{N-1} F_{N,i}^k a^{N-k}, \qquad i = 1, 2, 3, \cdots.$$

Proof. The case $i = 1$ amounts to $F_{N,1}^k = 1$, $k = 0, 1, \cdots, N-1$. If the theorem is true for some $i \geq 1$, then

$$a^{N+i+1} = \sum_{k=0}^{N-1} F_{N,i}^k a^{N-k+1} = \sum_{k=0}^{N-2} F_{N,i}^{k+1} a^{N-k} + F_{N,i}^0 a^{N+1} = \sum_{k=0}^{N-2} (F_{N,i}^{k+1} + F_{N,i}^0) a^{N-k} + F_{N,i}^0 .$$

Now

$$F_{N,i}^{k+1} + F_{N,i}^0 = F_{N,i+k+1} - \sum_{j=0}^{k} F_{N,i+j} + F_{N,i} = F_{N,i+1+k} - \sum_{j=0}^{k-1} F_{N,i+1+j} = F_{N,i+1}^k .$$

Also $F_{N,i} = F_{N,i+1}^{N-1}$, so the above equation reduces to

$$a^{N+i+1} = \sum_{k=0}^{N-1} F_{N,i+1}^k a^{N-k} .$$

REFERENCES

1. L. Carlitz, R. Scoville and V. E. Hoggatt, Jr., "Fibonacci Representations of Higher Order," *The Fibonacci Quarterly*, Vol. 10 (1972), pp. 43–70.
2. L. Carlitz, R. Scoville and V. E. Hoggatt, Jr., "Fibonacci Representations of Higher Order II," *The Fibonacci Quarterly*, Vol. 10 (1972), pp. 71–80.
3. L. Carlitz, R. Scoville and V. E. Hoggatt, Jr., "Fibonacci Representations," *The Fibonacci Quarterly*, Vol. 10 (1972), pp. 1–28.

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