It might be remarked that when $x = 1$, Eq. (5) becomes

$$Q_{n+3} = 2Q_{n+2} - Q_n \quad (n \geq 0)$$

which is a characteristic feature of the Fibonacci sequence of numbers.

[Setting $x = 1$ in $\{U_n\}$ and $\{T_n\}$ gives, on using (1) and (2) (or (3)), the sequences 1, 2, 3, 4, 5, 6, ⋯ and 2, 2, 2, 2, 2, 2, ⋯ , respectively.]

Further, one may notice that

(17) $$P_n = Q_n + F_{n-1} - 1,$$

where P_n are the numbers obtained from Jaiswal's polynomials $p_n(x)$ by putting $x = 1$, i.e., $P_n \equiv p_n(1)$.

$$(P_{n+1} = P_{n+1} + P_n - 1, \qquad P_0 = 1, \qquad P_1 = 1.)$$

Finally, $x = 1$ in (14) yields, with (16),

(18) $$F_{n+1} = \tfrac{1}{2}\left\{\sum_{r=0}^{[n/3]} \binom{n-2r}{r}(-1)^r 2^{n-3r} - \sum_{r=0}^{\left[\frac{n-3}{3}\right]} \binom{n-3-2r}{r}(-1)^r 2^{n-3-3r}\right\}.$$

Our results should be compared with the corresponding results produced by Jaiswal. The generating function (8), and the properties which flow from it such as (11) and (13), are slightly less simple than we might have wished. However, the Fibonacci property (16) could hardly be simpler. What we lose on the swings we gain on the roundabouts!

REFERENCE

1. D. V. Jaiswal, "On Polynomials Related to Tchebichef Polynomials of the Second Kind," *The Fibonacci Quarterly*, Vol. 12, No. 3 (Oct. 1974), pp. 263–265.

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Proposed by Guy A. R. Guillot, Montreal, Quebec, Canada.

Show that

(a) $$\frac{\pi}{2} = \sum_{n=1}^{\infty} \tan^{-1} \frac{2F_{2n+1}}{F_{2n}F_{2n+2}}$$

(b) $$\frac{\pi}{2} = \sum_{n=1}^{\infty} \cos^{-1} \frac{F_{2n}F_{2n+2}}{F_{2n}F_{2n+2}+2}$$

(c) $$\frac{\pi}{2} = \sum_{n=1}^{\infty} \sin^{-1} \frac{2F_{2n+1}}{F_{2n}F_{2n+2}+2}.$$

Proposed by Guy A. R. Guillot, Montreal, Quebec, Canada.

Find a function A_k in terms of k alone for the following expression.

$$F_n = \sum_{k=1}^{F_n} p_k - \sum_{k=1}^{F_n} A_k,$$

where p_k denotes the k^{th} prime and F_n denotes the n^{th} Fibonacci number.

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