\[
\sum_{n=1}^{p-1} p_n^{p^n} = 0, \quad \sum_{n=1}^{p-1} p_n^{-n} = 0.
\]

Multiplying the first equation with \(p^k\), the second with \(p^{-k}\), and using the easily verified formula

\[
\mu_n = \frac{(-1)^{n-1}}{\sqrt{3}}(p^n - p^{-n}),
\]

we get

\[
\sum_{n=1}^{p-1} (-1)^{n-1} \mu_{n+1} \frac{p_n}{p^{n+1}} = 0.
\]

Dividing by \(p\) and using

\[
\frac{1}{p} \left( \frac{p_n}{p^{n+1}} \right) \equiv \frac{(-1)^{n-1}}{n} \pmod{p}, \quad 1 \leq n \leq p - 1,
\]

we get the assertion.

*Also solved by Paul S. Bruckman.*

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(continued from page 288)

\(Z_\ell(t)\) represents the number of zeros of \(f_\ell\) which are \(\varepsilon\)-close to \(n_\ell\). By invariance of the complex integral, the functions \(Z_\ell(t)\) are constant since the functions \(f_\ell\) vary continuously and do not vanish on the path of integration. Hence, \(Z_\ell(0) = Z_\ell(1)\) for each \(\ell\). This says that in a small neighborhood of each zero of \(f_\ell\), there is a one-to-one correspondence of zeros of \(f_\ell\) with zeros of \(f_0\), in the required manner. \(\square\)

In the case of our given functions, we find that the zeros of the polynomial \(f_\ell(z)\) are close to the zeros of \(\varphi_\ell(z)\), which lie on the circle \(|z| = \alpha\), as required, and the zeros of \(f_\ell\) get closer to the circle as \(\ell \to \infty\). \(\square\)

*Also solved by P. Bruckman, O. Brugia & P. Filipponi, L. Kuipers, and the proposer.*

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