REDUCED AND AUGMENTED AMICABLE PAIRS TO 10^8

Walter E. Beck*
Department of Computer Science, University of Northern Iowa, Cedar Falls, IA 50614-0507

Rudolph M. Najar
Department of Mathematics, California State University, Fresno, Fresno, CA 93740-0108
(Submitted October 1991)

1. PRELIMINARIES

A *reduced amicable pair* is a pair of natural numbers, m and n, such that

$$m = \sigma(n) - n - 1; \quad n = \sigma(m) - m - 1,$$

where σ is the sum of divisors function. Jerrard and Temperley [4] studied numbers k satisfying $k = \sigma(k) - k \pm 1$ which they named *almost perfect numbers.* Lal and Forbes [5] first studied reduced amicable pairs and discovered nine pairs with smaller number $\leq 10^5$. (They coined the name "reduced amicable pair.") In an earlier paper [1], we extended the search to pairs with smaller number $\leq 10^6$, finding six new pairs. Hagis and Lord [3] extended the list to 10^7, discovering thirty-one new pairs, including two missed in [1]. The present paper extends the listing to 10^8. The paper [1] included a study of pairs m and n satisfying

$$m = \sigma(n) - n + 1; \quad n = \sigma(m) - m + 1,$$

called *augmented amicable pairs* and listed all pairs with smaller number less than 10^6. There were nine plus two other pairs both of whose elements exceeded one million. These arose from iterating the function $s_+(n) = \sigma(n) - n + 1$ on integers less than one million. A computer search extended the list to all pairs with smaller number less than one hundred million. Table 2 lists the pairs with one element less than ten million, except for powers of 2. Powers of 2 are fixed points of s_+ and are not included here. A complete list of the 84 pairs up to 10^8 is available from either author. The searches were carried out on the CRAY Y-MP at the University of Illinois at Urbana-Champaign, on Sun 4 work stations at the University of Northern Iowa, and on NeXT and Macintosh IIci stations at California State University, Fresno. Over half the search was run twice, once each at the latter two institutions.

* The first author was partially supported by National Center for Supercomputing Applications under TRA910033N and utilized the CRAY Y-MP system at the National Center for Supercomputing Applications, University of Illinois at Urbana-Champaign.

2. TABLES OF REDUCED AND AUGMENTED AMICABLE PAIRS

The tables of reduced and augmented amicable pairs follow.

TABLE 1. Reduced Amicable Pairs from 10^7 to 10^8

1.	12146750 = 5(3).7.11.631;	16247745 = 3(2).5.127.2843
2.	12500865 = 3(3).5.13.17.419;	12900734 = 1.7.11.19.4409
3.	13922100 = 2(2).3(2).5(2).31.499;	31213899 = 3(2).1549.2239
4.	14371104 = 2(5).3.11.31.439;	28206815 = 5.7.13.47.1319
5.	22013334 = 2.3(2).7.17.43.239;	37291625 = 5(3).7.17.23.109
6.	22559060 = 2(2).5.47.103.233;	26502315 = 3.5.7.83.3041
7.	23379224 = 2(3).11.23.11551;	26525415 = 3.5.7(2).151.239
8.	23939685 = 3(3).5.7(3).11.47;	31356314 = 2.11.23.31.1999
9.	26409320 = 2(3).5.7.257.367;	41950359 = 3(3).11.127.1031
10.	27735704 = 2(3).17.109.1871;	27862695 = 3(2).5.7.197.449
11.	28219664 = 2(4).11.109.1471;	32014575 = 3(3).5(2).43.1103
12.	33299000 = 2(3).5(3).7.67.71;	58354119 = 3(2).29.47.67.71
13.	34093304 = 2(3).97.31.41.479;	43321095 = 3(3).5.223.1439
14.	37324584 = 2(3).3(3).11.23.683;	80870615 = 5.7.17.199.683
15.	40818855 = 3.5.7.11.59.599;	42125144 = 2(3).23.179.1279
16.	41137620 = 2(2).3.5.17.31.1301;	84854315 = 5.7.13.251.743
17.	49217084 = 2(2).7.47.149.251;	52389315 = 3(3).5.11.35279
18.	52026920 = 2(3).5.11.23.53.97;	85141719 = 3(3).13.107.2267
19.	52601360 = 2(4).5.7.29.41.79;	97389039 = 3.11.17.173599
20.	61423340 = 2(2).5.11.23.61.199;	88567059 = 2.7(3).17.61.83
21.	62252000 = 2(5).5(3).79.197;	93423519 = 3(2).7.107.13859
22.	64045904 = 2(4).13.367.839;	70112175 = 3.5(2).7.83.1609
23.	66086504 = 2(3).11.750983;	69090615 = 3(2).5.11.29.4813
24.	66275384 = 2(3).7.17.43.1619;	87689415 = 3.5.11.179.2969
25.	68337324 = 2(2).3(3).11.23.41.61;	141649235 = 5.7.13.419.743
26.	72917000 = 2(3).5(3).13.71.79;	115780599 = 3(2).11.47.149.167
27.	76011992 = 2(3).7.179.7583;	87802407 = 3(3).7.11.157.269
28.	77723360 = 2(5).5.511.13.43.79;	145810719 = 3(3).41.107.1231
29.	89446860 = 2(2).3(2).5.17.29231;	197845235 = 5.7.17.332513
30.	93993830 = 2.5.7.727.1847;	99735705 = 3(5).5.23.43.83
31.	94713300 = 2(2).3(4).5(2).11.1063;	240536075 = 5(2).13.37.83.241
32.	94970204 = 2(2).7.107.31699;	96751395 = 3(3).5.13.29.1901
33.	97797104 = 2(4).19.23.71.197;	114332175 = 3(3).5(2).107.1583

Conjecture 0: There are infinitely many reduced (augmented) amicable pairs.

All pairs found have opposite parity. Since $\sigma(n) = m+n\pm 1 = \sigma(m)$, m and n have the same parity iff $\sigma(n) = \sigma(m)$ are odd iff odd prime factors in m and n occur only in even powers. Thus, we have

Conjecture 1: The numbers in a reduced (augmented) amicable pair are of opposite parity.

For each pair, consider the ratio k of the larger number divided by the smaller. In Table 1 the ratios range from 1.0045786 to 2.53962; in Table 2 from 1.0011028 to 2.64749. Thus,

Conjecture 2: For any $\beta > 0$, no matter how small, there exists a reduced (augmented) amicable pair such that $1 < k < 1 + \beta$.

TABLE 2. Augmented Amicable Pairs to 10^7

1.	6160 = 2(4).5.7.11;	11697 = 3.7.557
2.	12220 = 2(2).5.13.47;	16005 = 3.5.11.97
3.	23500 = 2(2).5(3).47;	28917 = 3(5).7.17
4.	68908 = 2(2).7.23.107;	76245 = 3.5.13.17.23
5.	249424 = 2(4).7.17.131;	339825 = 3.5(2).23.197
6.	425500 = 2(2).5(3).23.37;	570405 = 2.5.11.3457
7.	434784 = 2(5).3.7.647;	871585 = 5.11.13.23.53
8.	649990 = 2.5.11.19.311;	697851 = 3(2).7.11.19.53
9.	660825 = 3(3).5(2).11.89;	678376 = 2(3).19.4463
10.	1017856 = 2(11).7.71;	1340865 = 3(2).5.83.359
11.	1077336 = 2(3).3(2).13.1151;	2067625 = 5(3).7.17.139
12.	1238380 = 2(2).5.11.13.433;	1823925 = 3.5(2).83.293
13.	1252216 = 2(3).7.59.379;	1483785 = 3(3).5.29.379
14.	1568260 = 2(2).5.19.4127;	1899261 = 3(3).7.13.773
15.	1754536 = 2(3).7.17.19.97;	2479065 = 3.5.29.41.139
16.	2166136 = 2(3).7.47.823;	2580105 = 3.5.11.19.823
17.	2362360 = 2(3).5.7.11.13.59;	4895241 = 3.13.31.4049
18.	2482536 = 2(3).3.7(2).2111;	4740505 = 5.7(2).11.1759
19.	2537220 = 2(2).3.5.7(2).863;	5736445 = 5.11.13.71.113
20.	2876445 = 3(3).5.11.13.149;	3171556 = 2(2).19.29.1439
21.	3957525 = 3(3).5(2).11.13.41;	4791916 = 2(2).41.61.479
22.	4177524 = 2(2).3.13.61.439;	6516237 = 3.7.13.23869
23.	4287825 = 3(2).5(2).17.19.59;	4416976 = 2(4).59.4679
24.	5224660 = 2(2).5.7.67.557;	7524525 = 3.5(2).41.2447
25.	5559510 = 2.3.5.11.17.991;	9868075 = 5(2).7.17.31.107
26.	5641552 = 2(4).7.17.2963;	7589745 = 3(2).5.227.743
27.	5654320 = 2(4).5.7.23.439;	10058961 = 3.11.19.61.263
28.	5917780 = 2(2).5.11.37.727;	8024877 = 3(2).7(2).31.587
29.	6224890 = 2.5.7.17.5231;	7336455 = 3.5.7.107.653
30.	6274180 = 2(2).5.11.19(2).79;	9087741 = 3(3).13.17.1523
31.	6711940 = 2(2).5.17.19.1039;	9012861 = 3(2).11.13.47.149
32.	7475325 = 3.5(2).11.13.17.41;	8273668 = 2(2).13.107.1487
33.	7626136 = 2(3).7.43.3167;	9100905 = 3.5.11.19.2903
34.	7851256 = 2(3).7.19.47.157;	10350345 = 3.5.19.23.1579
35.	7920136 = 2(3).7.233.607;	9152505 = 3(2).5.23.37.239
36.	9026235 = 3(5).5.17.19.23;	9843526 = 2.7.11.41.1559

3. THE UNITARY CASE

In [2] searches for the unitary analogues of reduced and augmented amicable pairs to 10^5 were reported. Except for trivial cases, none were found. The search has been extended to 10^6 with no new results.

REFERENCES

1. W. Beck & R. M. Najar. "More Reduced Amicable Pairs." *Fibonacci Quarterly* **15.4** (1977): 331-32.
2. W. Beck & R. M. Najar. "Fixed Points of Certain Arithmetic Functions." *Fibonacci Quarterly* **15.4** (1977):337-42.
3. P. Hagis, Jr., & G. Lord. "Quasi-Amicable Numbers." *Math. Comp.* **31** (1977):608-11.
4. R. P. Jerrard & N. Temperley. "Almost Perfect Numbers." *Math. Magazine* **46** (1973):84-87.
5. M. Lal & A. Forbes. "A Note on Chowla's Function." *Math. Comp.* **25** (1971):923-25.

AMS numbers: 11A25; 11A99; 11B99

❖❖❖