A LOGARITHMIC FORMULA FOR FIBONACCI NUMBERS

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If the logarithm of the Fibonacci number F_n is plotted against n, it can be seen that for large n the graph is a straight line. Thus one might expect that Fibonacci numbers could be computed from a formula of the form

$$\log F_n = m n + b \quad , \tag{1}$$

where m is the slope of the line and b its intersection with the vertical axis. That this is so can easily be shown by manipulating Binet's formula

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right] \tag{2}$$

into the following form:

$$F_n = \frac{1}{5} \left(\frac{1+\sqrt{5}}{2} \right)^n \left[1 - \left(\frac{1-\sqrt{5}}{1+\sqrt{5}} \right)^n \right] \tag{3}$$

For large n, the second term within the bracket becomes negligible, and hence (3) becomes

$$F_n \quad \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n \tag{4}$$

Taking logarithms then gives

$$\log F_n \quad n[\log (1+\sqrt{5}) - \log 2] - \frac{1}{2} \log 5 \tag{5}$$

which is of the form (1). If base 10 is used, the characteristic of the logarithm computed in (5) then gives the order of magnitude of F_n. This knowledge is useful in determining required sizes of registers when setting up Fibonacci problems for computation.

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