PERFECT NUMBER "ENDINGS"

J.A.H. Hunter
Toronto, Ontario, Canada

Helen A. Merrill, in her book Mathematical Excursions, (Dover Publications, Inc.; New York) outlined the main features of Perfect Numbers. After stating that 6, 28, and 396 are Perfect Numbers, she continued:

> "The next Perfect Number is 8128, and the next contains eight digits. All these Perfect Numbers end in 6 or 28, but no one knows whether this is true of all such numbers."

The 1st edition of that book was published in 1933. It is possible that an elementary proof for the noted "endings" has been published somewhere since then, but if so it has escaped my notice.

Accordingly I venture to lay out the necessary but quite elementary proof that all even Perfect Numbers end in 6 or 28; the non-existence of any odd Perfect Number has not yet been proven.

Every even Perfect Number is known to be of general form: $2^{n-1}(2^n - 1)$.

The mod 100, by actual calculation for successive values of n ($n > 1$), we see that $2^{n-1}(2^n - 1)$ has successive values 6, 28, 0, 6; this sequence of 4 values being repeated for all cases up to $n = 22$. Still to mod 100, each of 2^{n-1} - $2^n - 1$ has a period of 20, repeating the remainders. Hence, the "6, 28, 0, 6" sequence of endings for the product $2^{n-1}(2^n - 1)$ must continue for all values of n.

It will be noted, the proof being trivial, that zero endings occur only when $n = 4k$.

Now, the actual particular form for all even Perfect Numbers requires n to be prime. Hence, with $n = 4k$, we can have no Perfect Number.

So, for all even Perfect Numbers we must have 6 or 28 as "ending."

xxxxxxxxxxxxxx