\[(L_{k+2}L_{k+3} - L_kL_{k+1}) = L_{k+2}(L_{k+2} + L_{k+1}) - L_{k+1}(L_{k+2} - L_{k+1})\]
\[= L_{k+2}^2 + L_{k+1}^2 = (F_{k+3} + F_{k+1})^2 + (F_{k+2} + F_k)^2\]
\[= (F_{k+2} + 2F_{k+1})^2 + (2F_{k+2} - F_{k+1})^2\]
\[= 5(F_{k+2}^2 + F_{k+1}^2) = 5F_{2k+3}\]
\[= 2F_{2k+3} + (F_{2k+5} - F_{2k+4}) + (F_{2k+2} + F_{2k+1}) + F_{2k+3}\]
\[= (F_{2k+2} + F_{2k+1}) + (F_{2k+5} + F_{2k+3})\]
\[= L_{2k+2} + L_{2k+4} = C\]

Thus, from (2) and (3) we have,

\[A^2 + B^2 = C^2 .\]

Also solved by J. A. H. Hunter and A. G. Shannon.

\[***\]

[Continued from p. 285]

RECURRING SEQUENCES — LESSON 1

ANSWERS TO PROBLEMS

1. \(a_n = n(n + 1)\); \(T_{n+3} = 3T_{n+2} - 3T_{n+1} + T_n\)
2. \(a_n = 3n - 2\); \(T_{n+2} = 2T_{n+1} - T_n\)
3. \(a_n = n^3\); \(T_{n+4} = 4T_{n+3} - 6T_{n+2} + 4T_{n+1} - T_n\)
4. \(T_{m+k} = 1, 3, 3, 1, 1/3, 1/3\), for \(k = 1, 2, 3, 4, 5, 6\), respectively
5. \(T_{n+1} = \sqrt{1 + T_n^2}\)
6. \(T_{n+3} = 3T_{n+2} - 3T_{n+1} + T_n\)
7. \(T_{n+1} = aT_n\)
8. \(T_{n+3} = 3T_{n+2} - 3T_{n+1} + T_n\)
9. \(T_{2n} = a, \ T_{2n} = 1/a\)
10. \(T_{n+1} = 1/(2 - T_n)\)

\[***\]