Recent developments in free actions of fine groups on homotopy spheres

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P.A. Smith proved that if a finite group G acts freely on a sphere then all of its abelian subgroups must be cyclic. This condition in known to be equivalent to the *periodicity* of G and such groups have been classified by Suzuki-Zassenhaus. Then, R.G. Swan showed that a converse of Smith's result does hold for homotopy spheres.

Let G be a finite group with period $2d$ and $X(n)$ an n-dimensional CW-complex with the homotopy type of the n-sphere $\mathbb{S}^n$. On the base of a number of papers below, we compute the number distinct homotopy types of orbit spaces $X(2dn-1)/\mu$ with respect to free and cellular G-actions μ on all $X(2dn-1)$ and determine the groups $\mathcal{E}(X(2dn-1)/\mu)$ of self-homotopy equivalences. These groups depend on the group G only.

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