Relative EHP sequence

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My talk is expository. Let B be the mapping cone of a map $\varphi : A \to X$. Then, Toda(1956) defined a map $T : \Omega(B, X) \to \Omega^2(\Sigma A \wedge B)$ where $\Omega(B, X)$ is the homotopy fiber of the inclusion map $X \to B$. Boardman-Steer(1967) and Ganea(1968), independently, constructed a map $\mu : \Omega B \to \Omega^2(\Sigma A \wedge B)$ using the coaction map of B. Boardman-Steer obtained a very useful result about the functional cup products. These maps, in fact, factor through $\Omega^2(\Sigma A \times B, \Sigma A \vee B)$, so they are "delicate Hopf invariants". I describe these construction and show that they are essentially the same and that these invariants have some useful properties related to the following James's old homotopy exact sequence(1954) for some range.

$$\pi_i(B, X) \xrightarrow{p'_*} \pi_i(\Sigma A) \xrightarrow{H_\varphi} \pi_i(\Sigma(X \wedge A)) \xrightarrow{\Delta} \pi_{i-1}(B, X) \to \cdots ,$$

here the above exact sequence can be regarded as "relative EHP-sequence".