Miller Spaces and Spherical Resolvability of Finite Complexes

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Abstract:

Finite CW complexes can be thought of as those spaces that can be built from spheres using the homotopy pushout (which is a kind of homotopy colimit) construction a finite number of times. I will show that the loop space of a (nilpotent) finite complex can be built from spheres using various homotopy *limits*.

We call a space M a *Miller space* if the space of pointed maps $\mathrm{map}_*(M, K)$ is (weakly) contractible for all nilpotent finite complexes K. It follows from the main theorem, that M is a Miller space if and only if $\mathrm{map}_*(M, S^n) \sim *$ for all $n \geq 1$.