Brain Teasers (Chase Report, May 2012)

SOLUTIONS

1. Are there two integers m, n such that $m^3 - n^3 = (2012)^2$?

   The **answer** is no! Working modulo 13, we have m^3 and n^3 must be congruent to 0, 1, 5, 8 or 12, and so $(2012)^2$ is congruent to $10 \neq m^3 - n^3 \pmod{13}$.

2. Find the next three numbers in the sequence

   4,6,9,10,14,15,21,22,25,26,33,34,35,3839,46,49,51,55…

   Answer: The numbers in the sequence share the common property that each is divisible by precisely two primes.

3. Is it possible to cut a diamond so every face of the diamond is a polygon with 7 sides?

   Answer: Euler's formula for a simple polyhedron states that $F - E + V = 2$, where F, E, V are, respectively, the number of faces, edges and vertices. If each face had 7 edges then, since each edge is an edge in exactly two faces, we must have $7F = 2E$. At the same time each face would have 7 vertices and each vertex lies on at least three faces, so $7F \geq V$. But then $(F - E + V) \leq F - 7F/2 + 7F/3 < 0$, contradicting Euler's formula. Hence no such diamond can be cut.

4. What is the probability that at least two of the next 24 Fields medal winners will have the same birthday?

   Answer: Ignoring the possibility of a birthday on Feb.29 we see that there are 365^{24} different ways in which the Fields Medalists' birthdays may be distributed. On the other hand, no two lie on the same day, then there would be only $365(365 - 1)\ldots(365 - 23)$ possible ways. Hence the required probability is $1 - [365(365 - 1)\ldots(365 - 23)/365.365. \ldots .365]$ which equals approximately 0.538344.

5. A regular hexagon and an equilateral triangle have the same perimeter. What is the ratio of their areas?
   Answer: Partition each figure into congruent triangles. Then by creating an equilateral triangle of four smaller triangles and a regular hexagon of six smaller triangles, we see easily that the required ratio is 3 : 2.

6. What letter, apart from E, can be added to the following to complete this sequence? S E Q U E N C _

   Answer: Place F on the underline so as to form the final E.

7. Sherlock Holmes was relaxing in his study, when suddenly a snowball struck one of the windows overlooking the street below, causing it to shatter. Quickly, he went over to the window to investigate, and looking out, just caught sight of the Willoughby triplets, Danny, Mark and Oliver, disappearing rapidly around the corner. The next morning he received the following anonymous message:
   ? Willoughby. I'm certain he broke your window.
   According to this, which one of the triplets should he question about the incident?

   Answer: Mark. Substitute "?" for Question Mark.

8. Which is the odd one out?
   A TOR
   B SHOP
   C RATE
   D RECTOR
   E SIGN
   F ANTIC

   Answer: Place letters as shown below to get six words:
   A CTOR
   B ISHOP
   C URATE
   D IRECTOR
   E NSIGN
   F RANTIC
   Now FRANTIC is the odd one out because all the others are occupations.