Buyer beware: A penny lost does add up over time

February 15, 2013 - 4:56pm **BY JASON I. BROWN**

The cups of pennies on my dresser seem to stare sadly back at me. Yes, it has already happened — purchases are being rounded to the nearest nickel, and the copper coins are on their way out. Not that I hold any particular affection for the coins, but the change in change is a change in itself.

The mathematics of the rounding intrigues me. Surely the loss or gain of a penny here or there is a trifling matter, n'est-ce pas? But over cup of coffee I mulled over the matter. There is something called the long tail of a distribution, the very far trail of very small amounts.

It might seem like a large number of very small numbers add up to little. For example, if you add up a half and a quarter and an eighth an so, forever, it does add up, but just to 1.

Here's one way to think of it. Imagine you have for Valentine's Day a chocolate cake in front of you. At first you eat half the cake, leaving half a cake. Then you eat half of the half that's left — one quarter of the cake, again leaving a quarter.

Then you eat half of what's left, an eighth of the cake, leaving an eighth, and so on. You leave less and less of the cake as time goes by, and eventually (after an infinite number of pieces) you have finished the cake (and have some explaining to do to your spouse).

On the other hand, sometimes, adding up small numbers add up to a lot. If instead of adding up ½, ¼, 1/8, we add up ½, 1/3, ¼, 1/5, we get a completely different story. Notice that 1/3 + ¼ is more than ¼ + ¼, which is ½, and the next 4 terms, 1/5+1/6+1/7+1/8 add up to more than four eighths, which is ½ again.

Proceeding, we find that adding up the fractions ½ , 1/3 , ¼ , 1/5 and so on is more than adding up ½ repeatedly, and so it gets as big a you like if you add up enough terms (though it will take awhile to grow).

So now back to the long tail and my cup of coffee. The cup of coffee cost $1.24, and with 15 per cent tax, that comes to 1.24 x 1.15 = 1.426, or one dollar and 43 cents. As I paid $1.50 cash, the proprietor gave me back a nickel, and kept the extra 2 cents.

Not much to make a fuss about, right? But what, I pondered, was the big tail of what an extra two cents means to the coffee franchise, when it's an extra two cents over the long tail of many, many coffees.

For example, with a million cups of coffee sold, that would amount to an extra $20,000 floating around. Not just chump change.

I only bought the small coffee, so I looked at the prices of the other sizes: $1.50, $1.68 and $1.90, and in all cases, the prices with tax rounded up, rather than down, to the nearest nickel, meaning some free pennies from the long tail for the coffee makers. You would think that perhaps by chance some of the numbers would round down, saving money for the purchasers, but it seems not.

Between the math and the coffee, that's enough to keep me up at night.

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